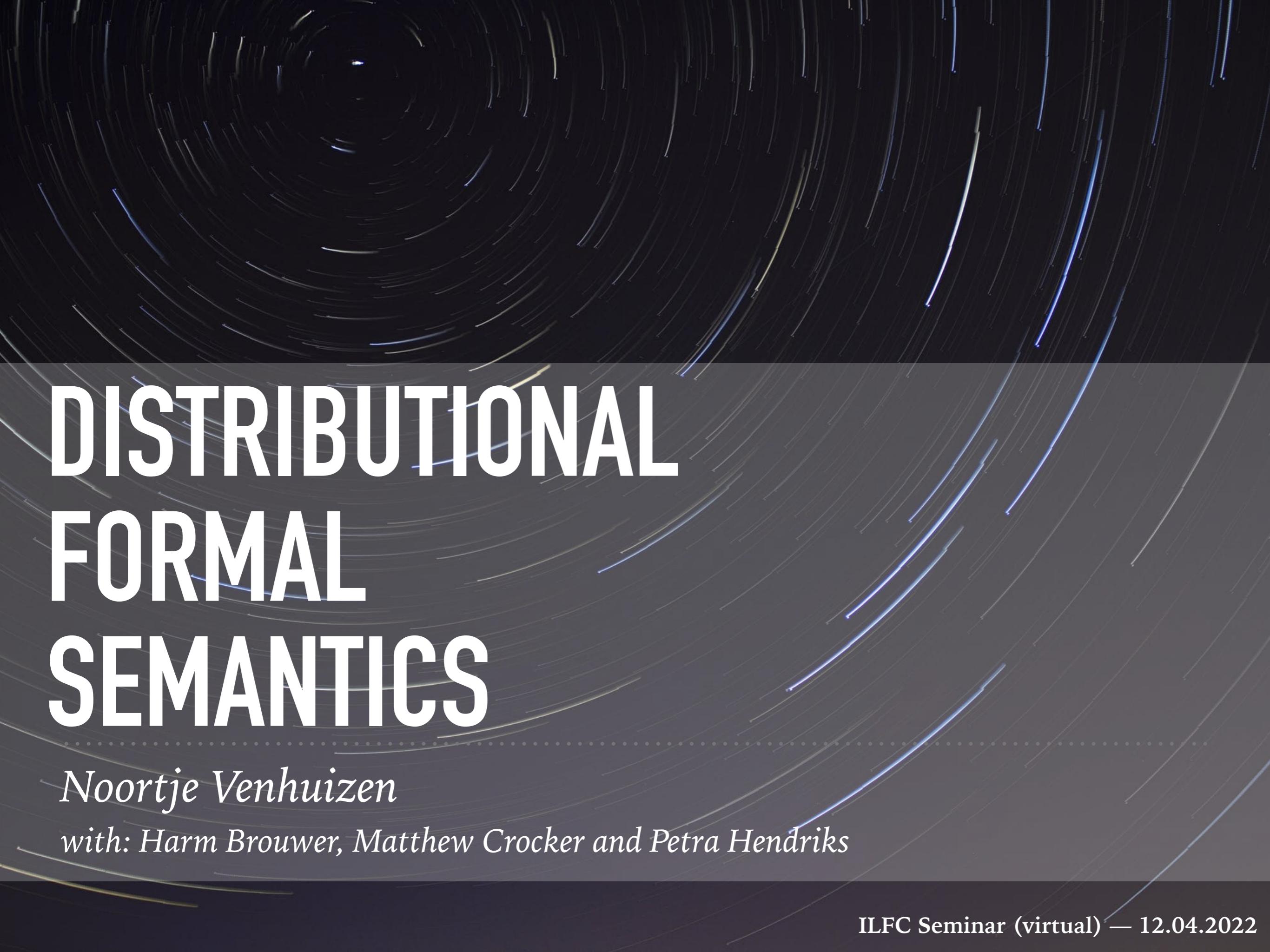


DISTRIBUTIONAL FORMAL SEMANTICS



Noortje Venhuizen

with: Harm Brouwer, Matthew Crocker and Petra Hendriks

NATURAL LANGUAGE SEMANTICS

Model-theoretic Semantics

- Truth-conditional meaning
- Logical entailment
- Compositionality

Distributional Semantics

- Semantic similarity
- Empirically driven
- Cognitively inspired

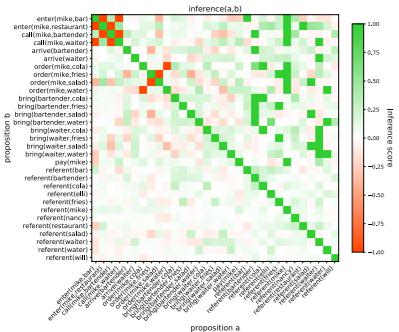
?

E.g., Baroni *et al.* (2010,2014); Boleda & Herbelot (2016); Coecke *et al.* (2010); Grefenstette & Sadrzadeh (2011); Socher *et al.* (2012)

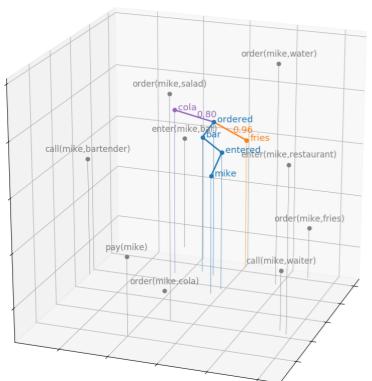
A FRAMEWORK FOR DISTRIBUTIONAL FORMAL SEMANTICS

	p^1	p^2	p^3	\dots
M_1	1	1	0	\dots
M_2	1	0	0	\dots
M_3	0	1	0	\dots
M_4	1	1	1	\dots
\dots	\dots	\dots	\dots	\dots

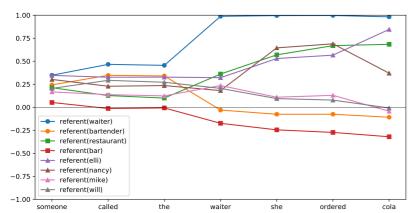
A meaning space for Distributional Formal Semantics



Formal properties of the meaning space

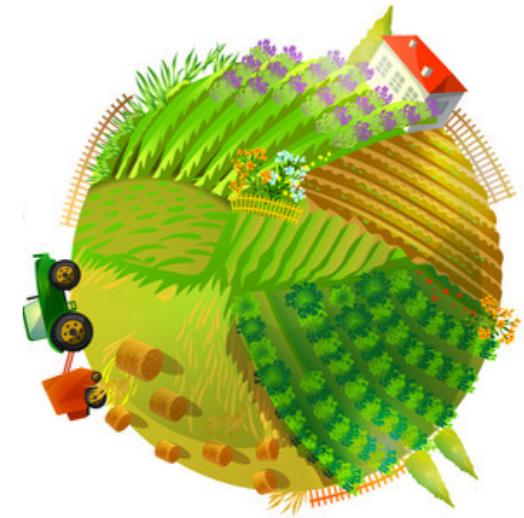


Incremental meaning construction



Semantic processing in the meaning space

FROM MODELS TO MEANING SPACE



...

$$M_1 = \langle U_1, V_1 \rangle$$

$$p_1 \wedge \neg p_2 \wedge p_3 \wedge \dots$$

$$M_2 = \langle U_2, V_2 \rangle$$

$$\neg p_1 \wedge p_2 \wedge p_3 \wedge \dots$$

$$M_3 = \langle U_3, V_3 \rangle$$

$$\neg p_1 \wedge p_2 \wedge \neg p_3 \wedge \dots$$

$$M_n = \langle U_n, V_n \rangle$$

$$\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge \dots$$

- The set of models $\mathcal{M}_{\mathcal{P}}$ — describing states-of-affairs over propositions in \mathcal{P} — defines a meaning space
- Propositional meaning defined by co-occurrence across models

CAPTURING THE STRUCTURE OF THE WORLD

“A boy rides a bike”

Boy is (likely) outside

Boy is not asleep

If it’s evening, the light is on

The bike has wheels

etc.



World knowledge restricts propositional co-occurrence in the meaning space derived from the set of models \mathcal{M}_P

- Hard world knowledge constraints restrict individual models
- Probabilistic constraints define probabilistic co-occurrences across the set of models \mathcal{M}_P

DFS MEANING SPACE $S_{\mathcal{M} \times \mathcal{P}}$

propositional meaning vectors

	p_1	p_2	p_3	p_4	\vdots
M_1	1	1	0	0	...
M_2	1	0	0	1	...
M_3	0	1	0	1	...
M_4	1	1	1	1	...
M_5	0	1	0	0	...
...

$$[\![p_j]\!]^{\mathcal{M}} := v(p_j)$$

where: $v_i(p_j) = 1$ iff $M_i \models p_j$

- **Incremental inference-based probabilistic sampling:** Based on a set of propositions \mathcal{P} , we sample a set of models $\mathcal{M}_{\mathcal{P}}$ —taking into account hard and probabilistic world knowledge constraints
- **Co-occurrence defines meaning:** Propositions with related meanings are true in many of the same models, resulting in similar meaning vectors

THE DISTRIBUTIONAL HYPOTHESIS REVISITED

“

You shall know a ~~word~~ *proposition*
by the company it keeps

- J. R. Firth (1957)

MEANING VECTOR COMPOSITION

Meaning vectors can be combined to define compositional meanings

- Standard logical operators interpreted as in model-theory

$$v_i(\neg p) = 1 \quad \text{iff } M_i \not\models p$$

$$v_i(p \wedge q) = 1 \quad \text{iff } M_i \models p \text{ and } M_i \models q$$

... etc.

- Quantification is defined relative to the combined universe of \mathcal{M}_P : $\mathcal{U}_{\mathcal{M}} = \{e_1 \dots e_m\}$ (thereby preserving entailment in \mathcal{M}_P)

$$v_i(\forall x \varphi) = 1 \quad \text{iff } M_i \models \varphi[x|e_1] \wedge \dots \wedge \varphi[x|e_m]$$

$$v_i(\exists x \varphi) = 1 \quad \text{iff } M_i \models \varphi[x|e_1] \vee \dots \vee \varphi[x|e_m]$$

PROBABILITIES IN THE MEANING SPACE

All (sub-)propositional meaning vectors inherently encode (co-)occurrence probabilities

- Prior probability of meaning vector a

$$P(a) = \frac{1}{|\mathcal{M}|} \sum_i \vec{v}_i(a)$$

- Conjunction probability between a and b

$$P(a \wedge b) = \frac{1}{|\mathcal{M}|} \sum_i \vec{v}_i(a) \vec{v}_i(b)$$

- Conditional probability of a given b

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

	p_1	p_2	p_3	p_4	
M_1	1	1	0	0	...
M_2	1	0	0	1	...
M_3	0	1	0	1	...
M_4	1	1	1	1	...
	0	1	0	0	...

QUANTIFYING PROBABILISTIC INFERENCE

Probabilistic logical inference of meaning vector a given b

$$\text{inference}(a,b) = \begin{cases} [P(a|b) - P(a)] / [1 - P(a)] & \text{if } P(a|b) > P(a) \\ [P(a|b) - P(a)] / P(a) & \text{otherwise} \end{cases}$$

- $P(a|b) > P(a)$: Positive inference (b increases probability of a)

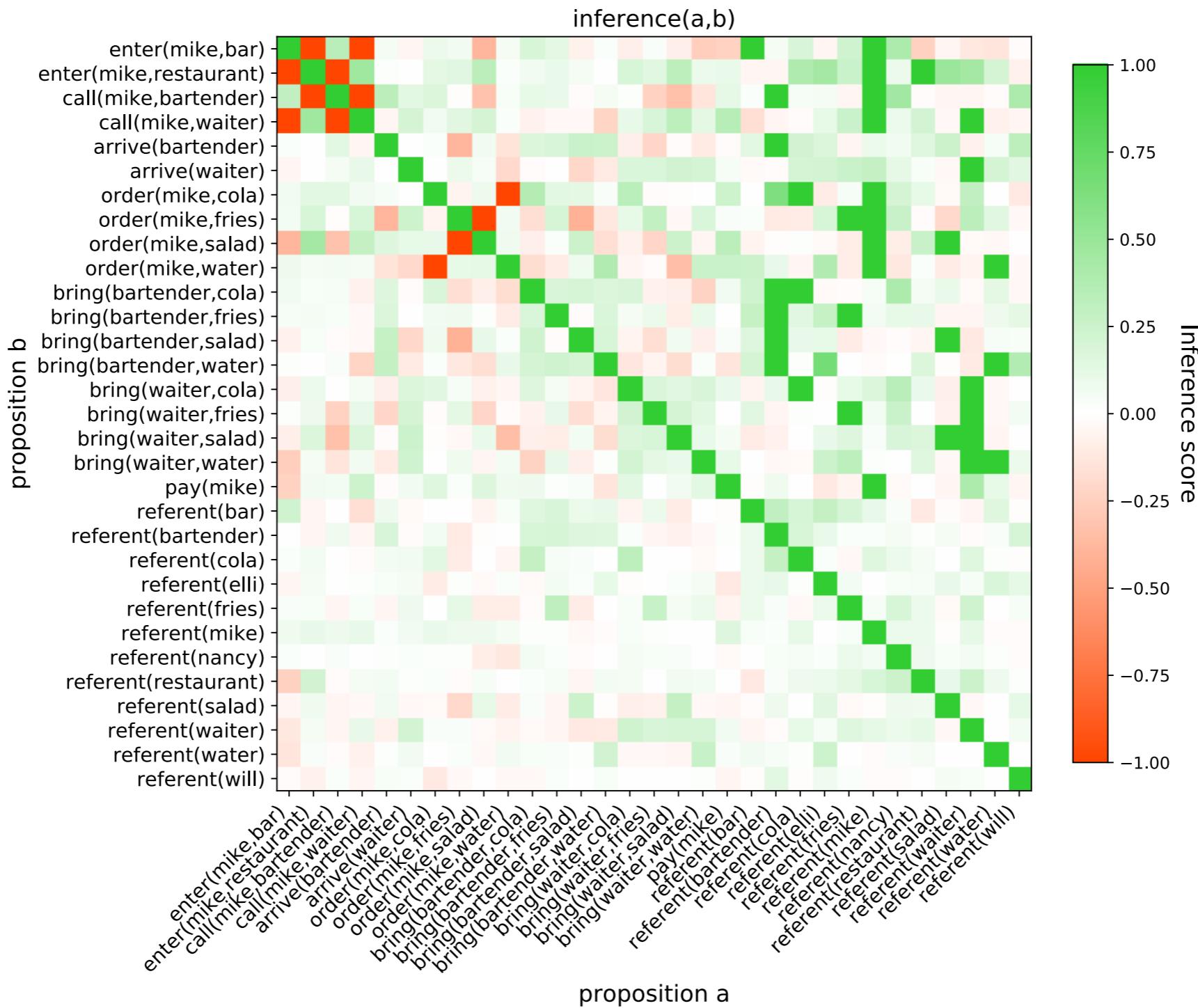
$$\text{inference}(a,b) = 1 \Leftrightarrow b \vDash a$$

- $P(a|b) \leq P(a)$: Negative inference (b decreases probability of a)

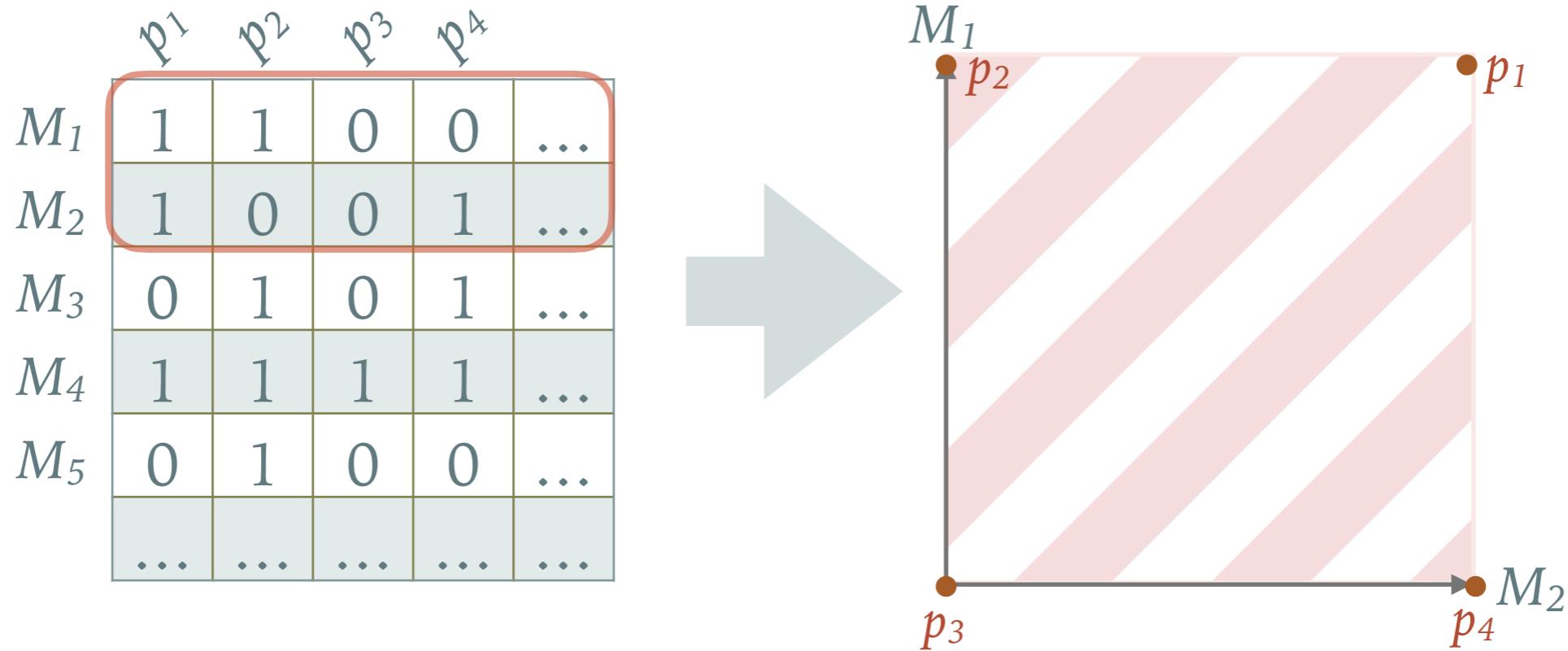
$$\text{inference}(a,b) = -1 \Leftrightarrow b \vDash \neg a$$

WORLD KNOWLEDGE IN THE MEANING SPACE

We sampled a meaning space of 150 models describing 51 propositions

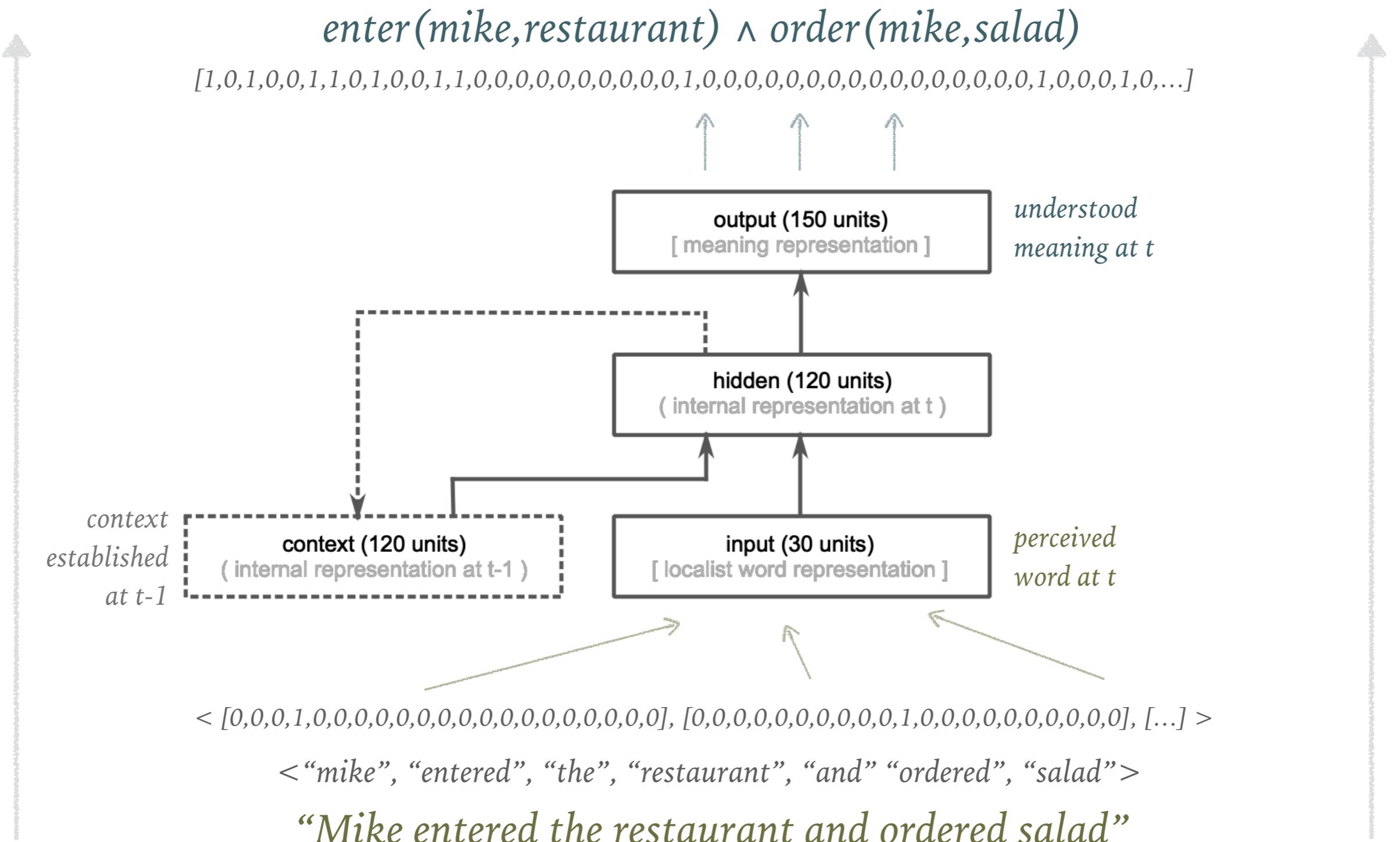


CONTINUOUS NATURE OF THE MEANING SPACE

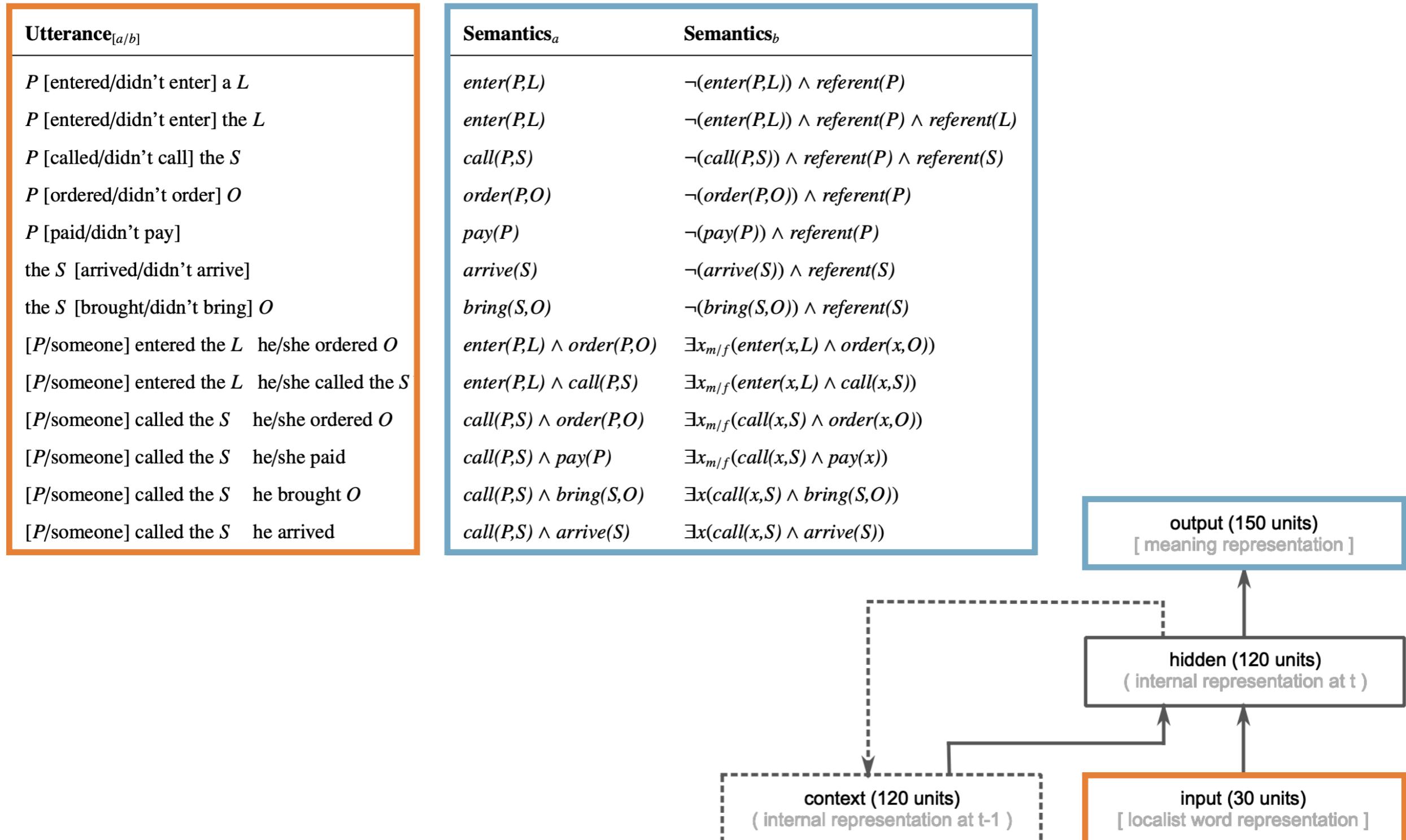


- Each point in the meaning space can be interpreted relative to \mathcal{M}_P
 - Binary vectors: propositional meanings (simple or complex)
 - Real-valued vectors: sub-propositional meanings
- Sub-propositional meaning derives from incremental mapping from (sequences of) words to proposition-level meanings (Frank et al., 2009, *Cognition*)

A MODEL OF INCREMENTAL MEANING CONSTRUCTION



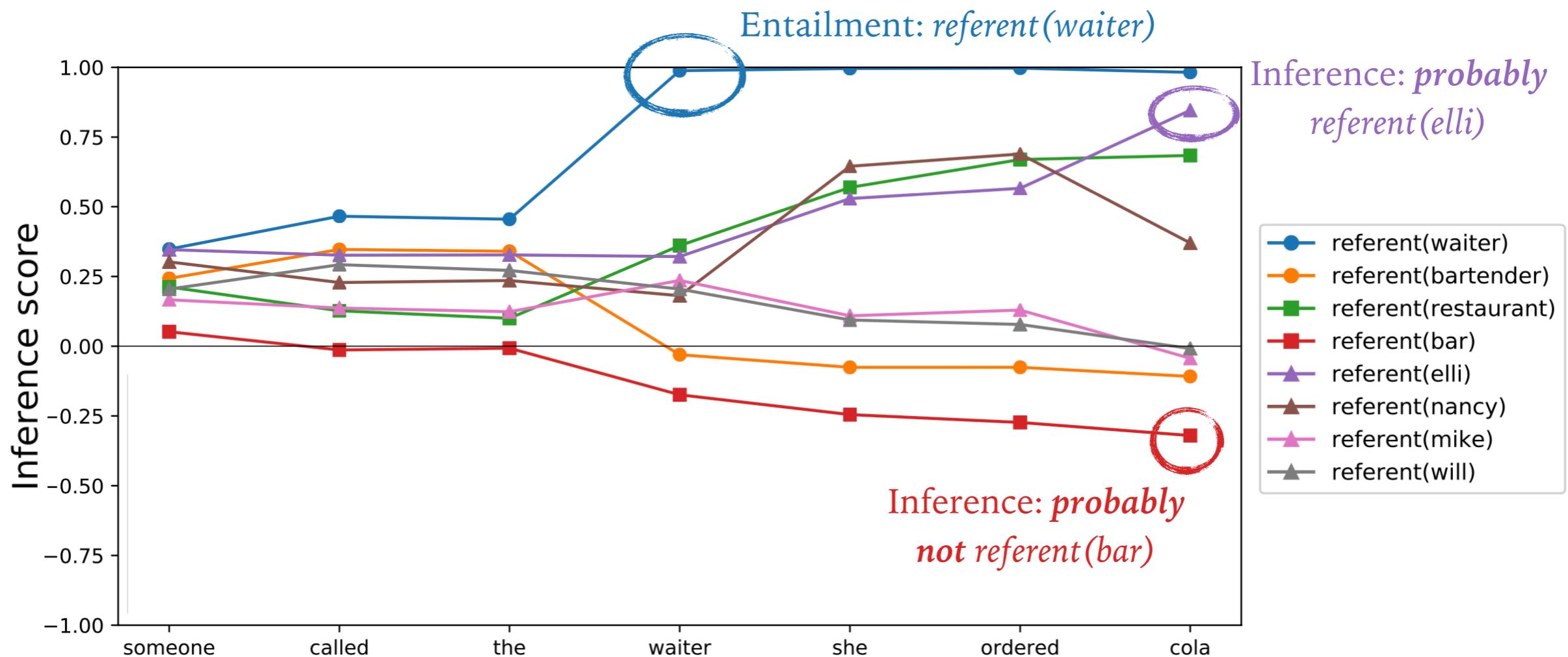
CONSTRUCTING THE MODEL: LANGUAGE



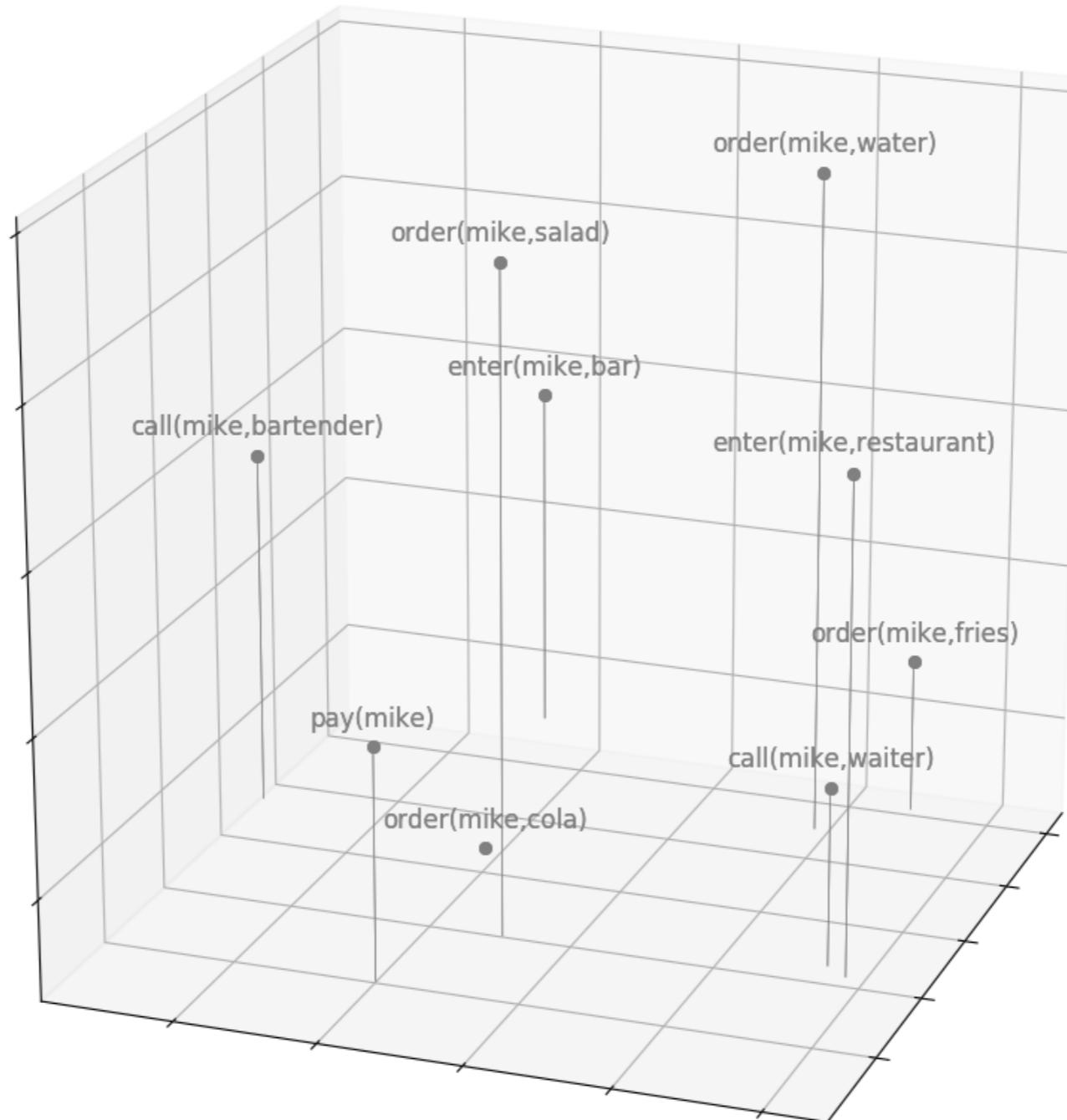
ENTAILMENT AND INFERENCE

Incremental meaning construction in the model is driven by:

- Sentence-semantics mappings (literal utterance meaning)
- Structure of the meaning space (probabilistic inferences)

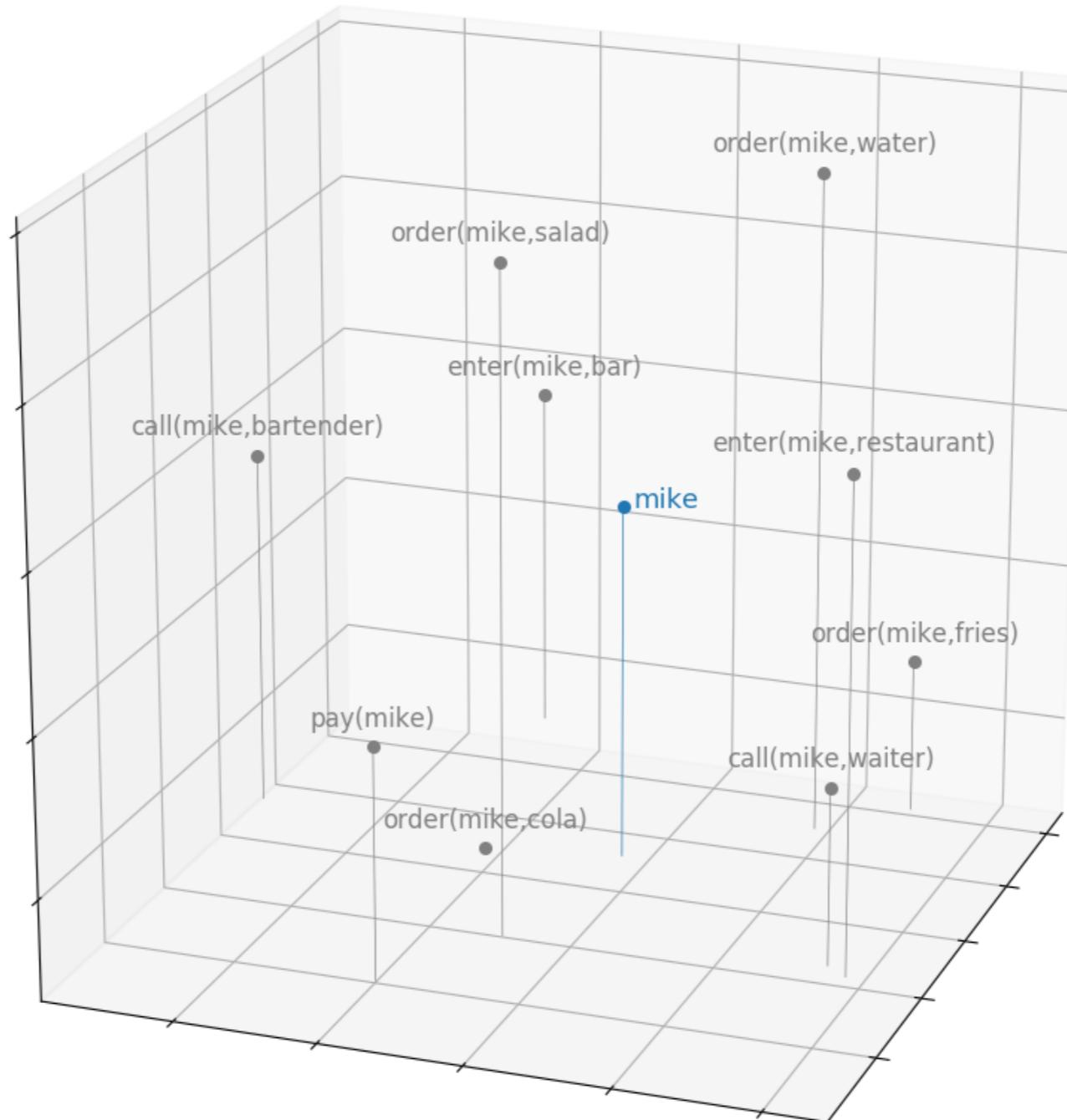


MEANING SPACE NAVIGATION



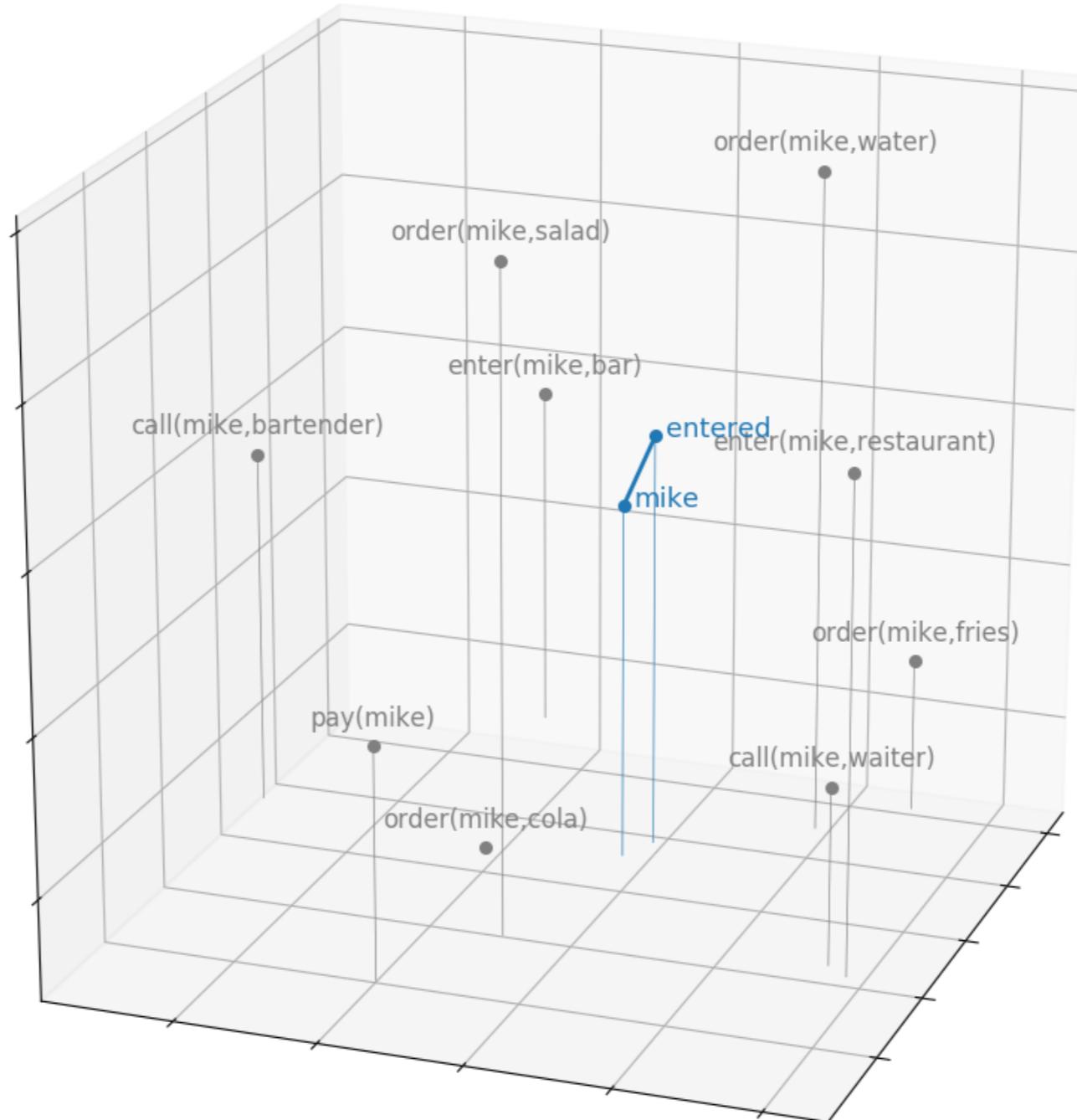
- Multi-dimensional scaling from 150 → 3 dimensions
(for a subset of the propositions)

MEANING SPACE NAVIGATION



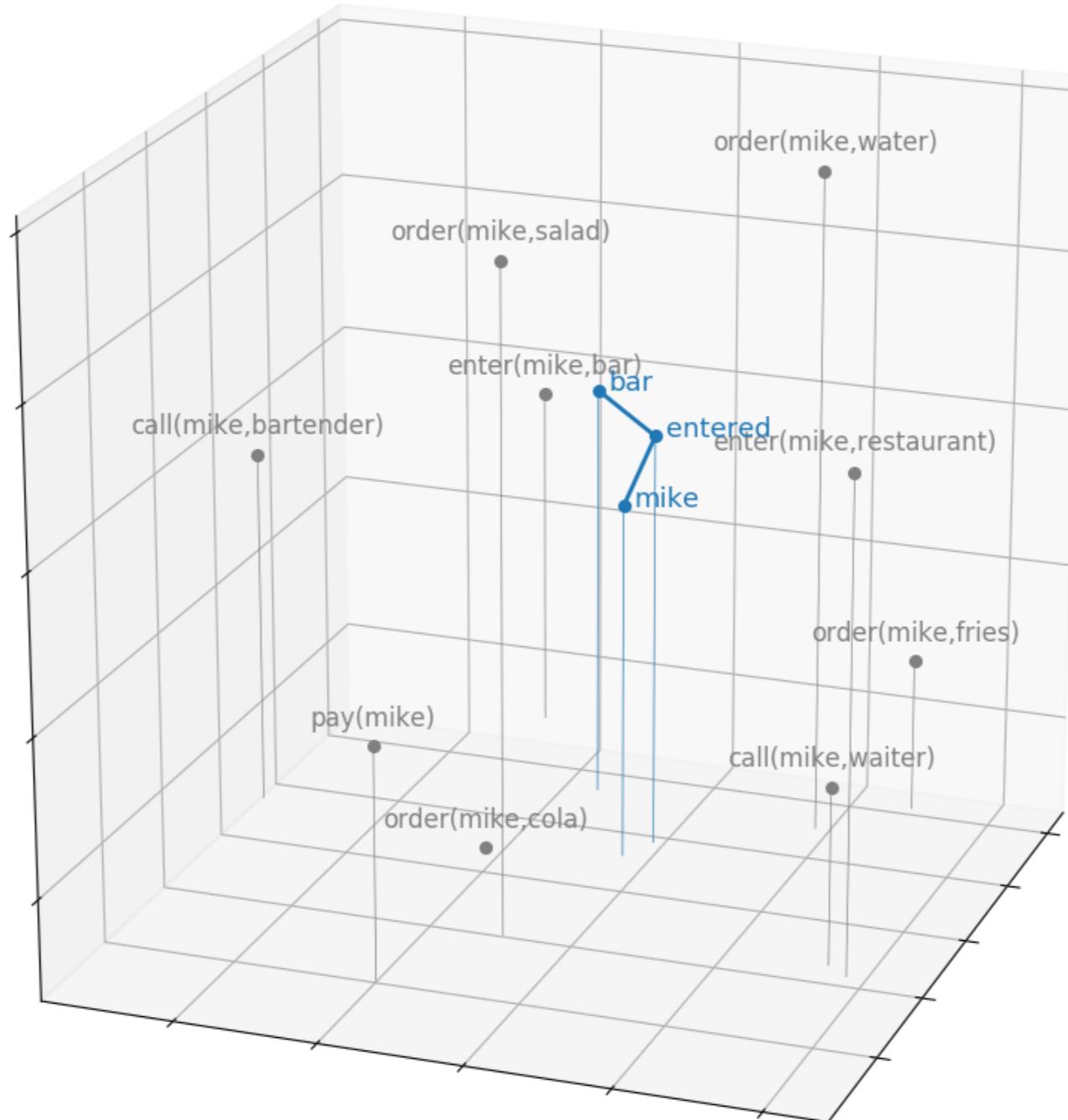
- Model-derived meaning of '*mike*'

MEANING SPACE NAVIGATION



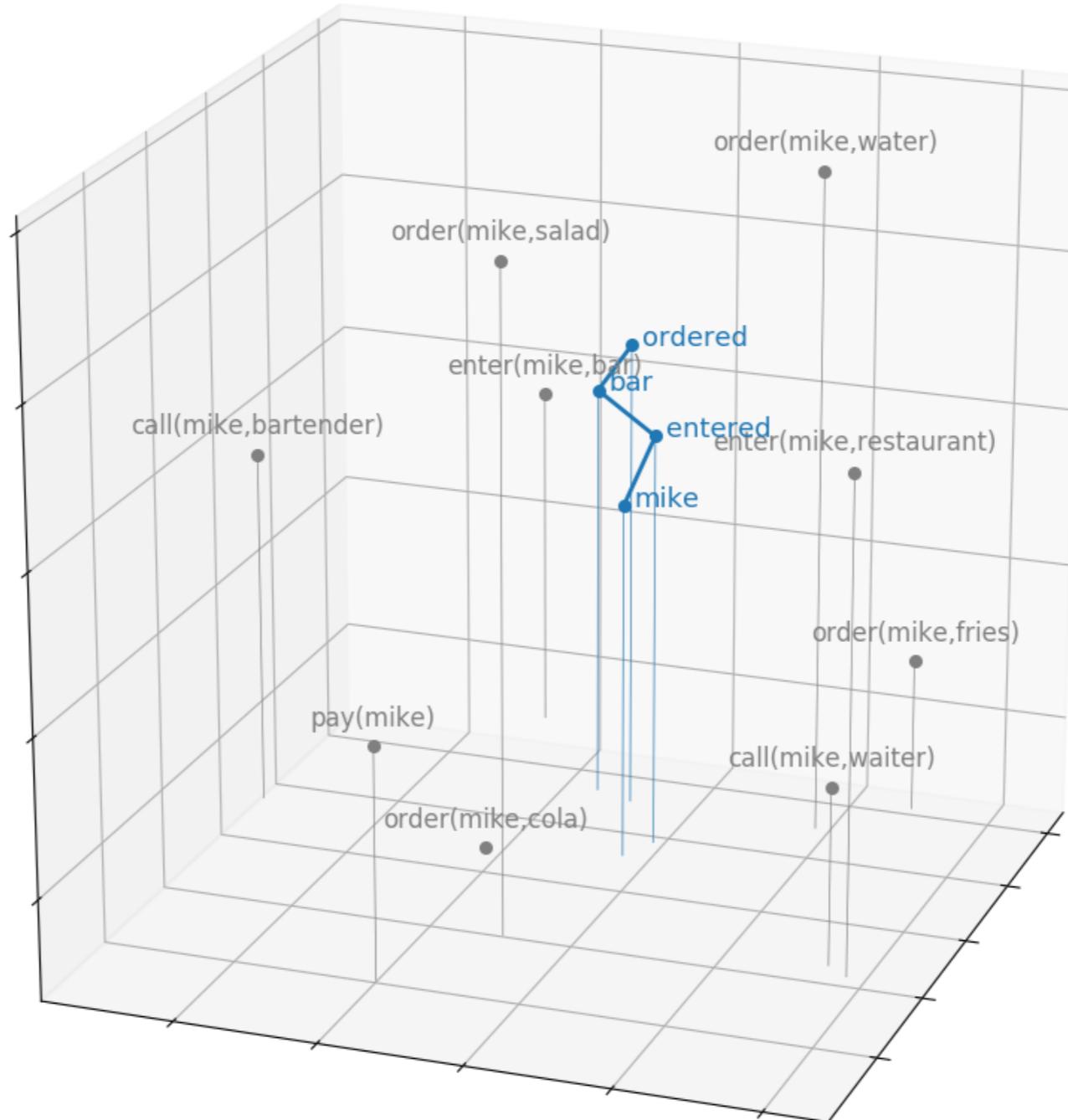
- Model-derived meaning of '*mike entered*'

MEANING SPACE NAVIGATION



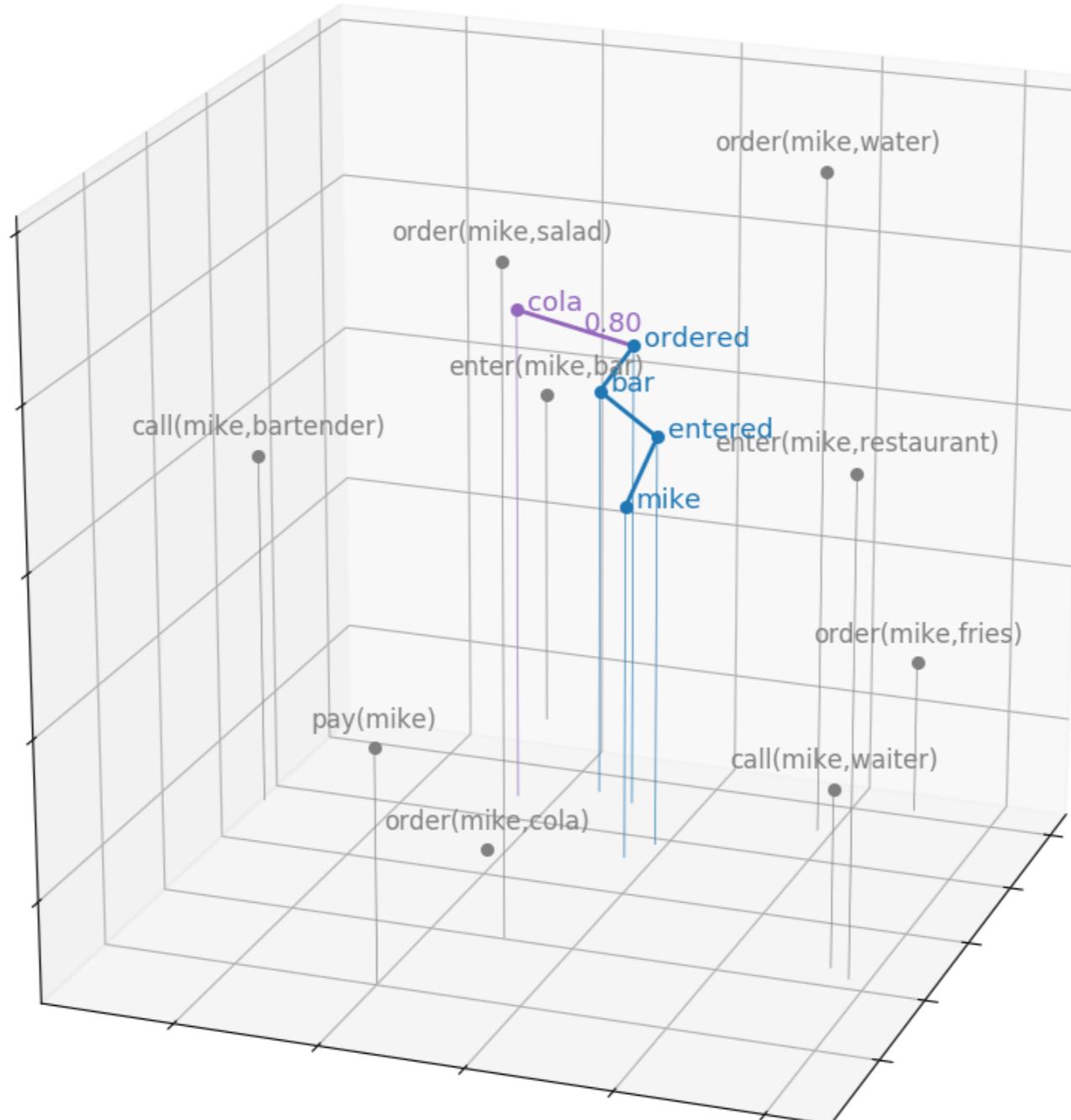
- Model-derived meaning of '*mike entered the bar*'

MEANING SPACE NAVIGATION



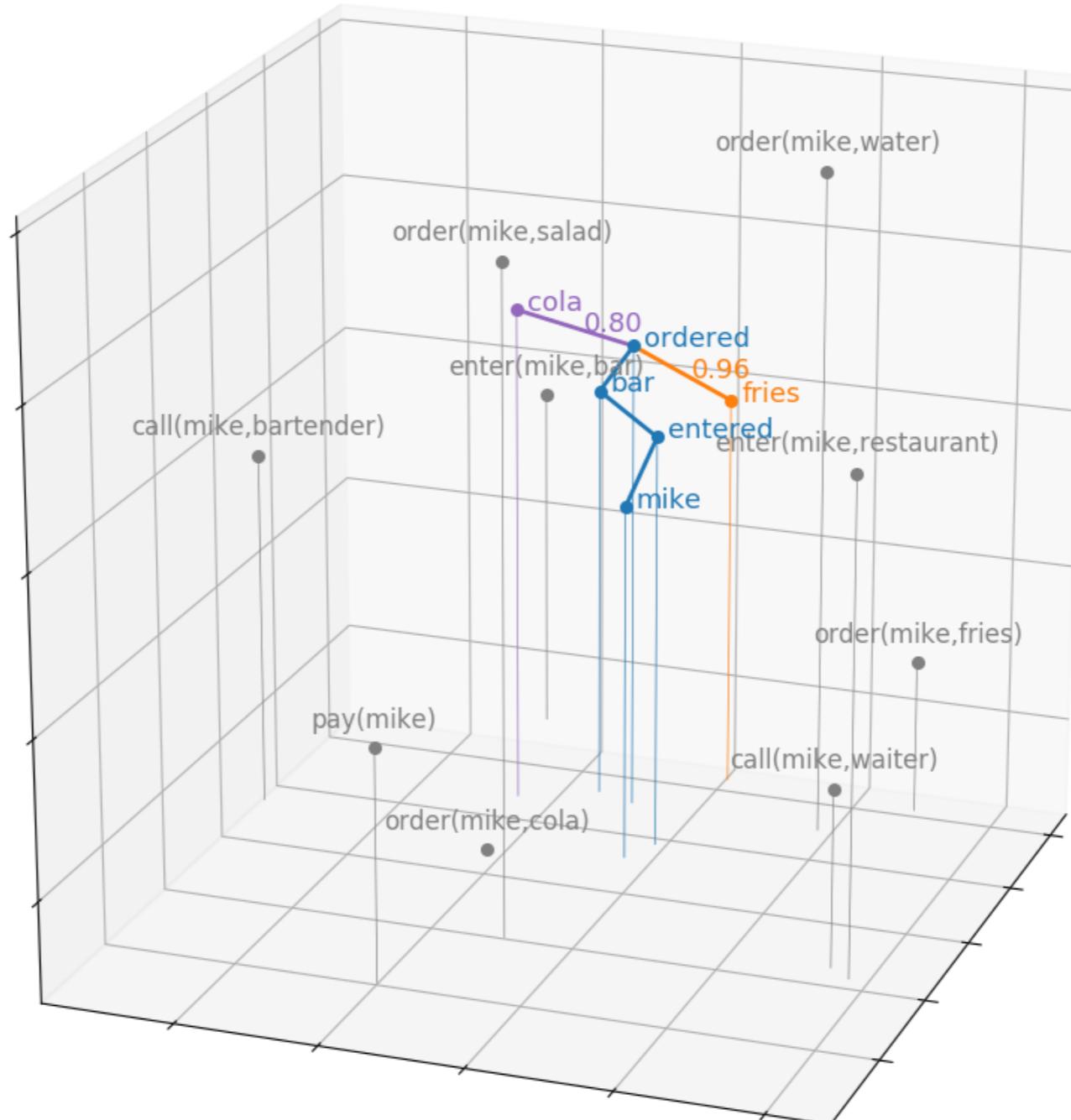
- Model-derived meaning of '*mike entered the bar [.] he ordered*'

MEANING SPACE NAVIGATION



- Model-derived meaning of '*mike entered the bar [...] he ordered cola*'
- Distance \propto Surprisal: Expected transition in meaning space

MEANING SPACE NAVIGATION



- Model-derived meaning of '*mike entered the bar [...] he ordered fries*'
- Distance \propto Surprisal: Unexpected transition in meaning space

INFORMATION THEORY IN DFS

Probabilistic nature of meaning space allows for defining formal notion of **information** (Shannon, 1948)

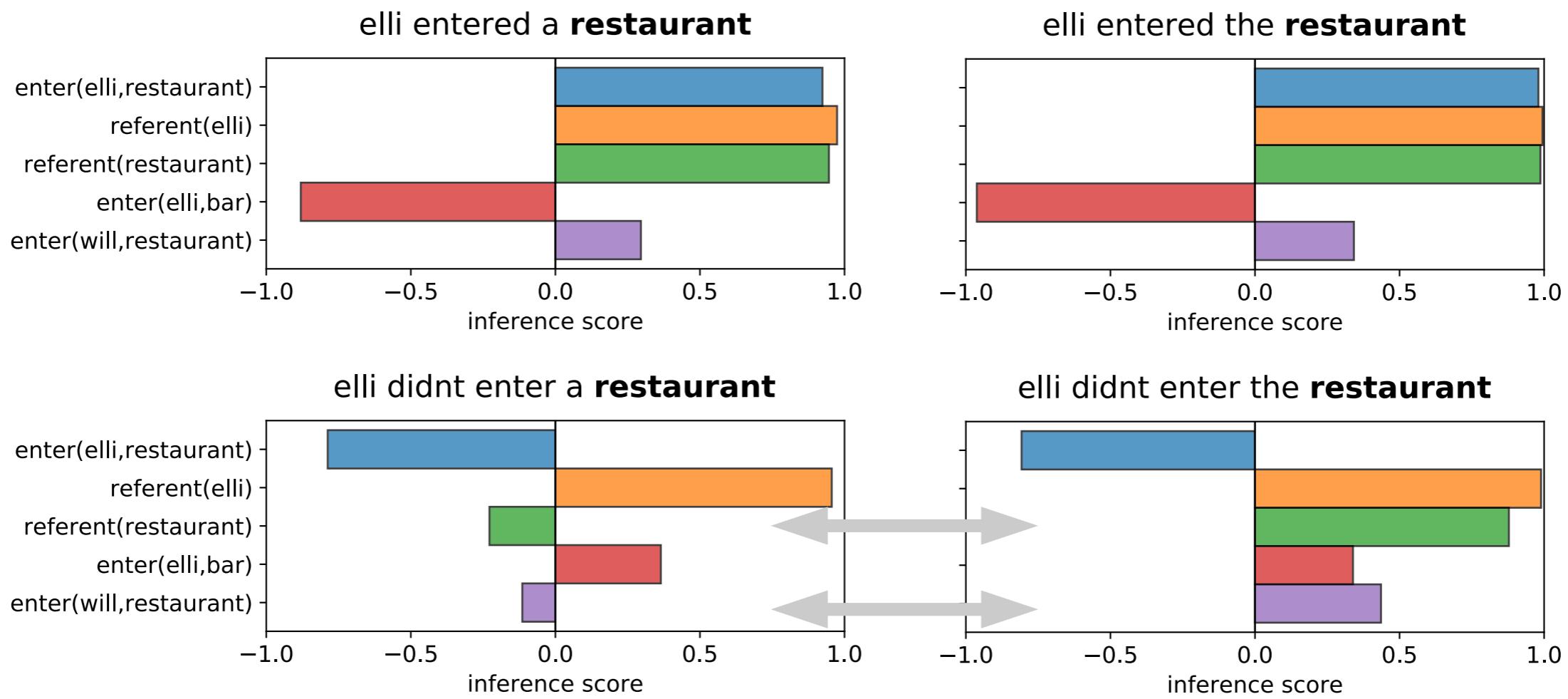
- **Surprisal** quantifies the expectancy of words in context
- Higher Surprisal \Leftrightarrow increased processing cost (Hale, 2001; Levy, 2008)
- In DFS, Surprisal quantifies expectancy of transition in meaning space, triggered by message m_{ab} :

$$S(m_{ab}) = -\log P(b | a)$$

→ Word-by-word information effects of semantic construction

NEGATION AND PRESUPPOSITION

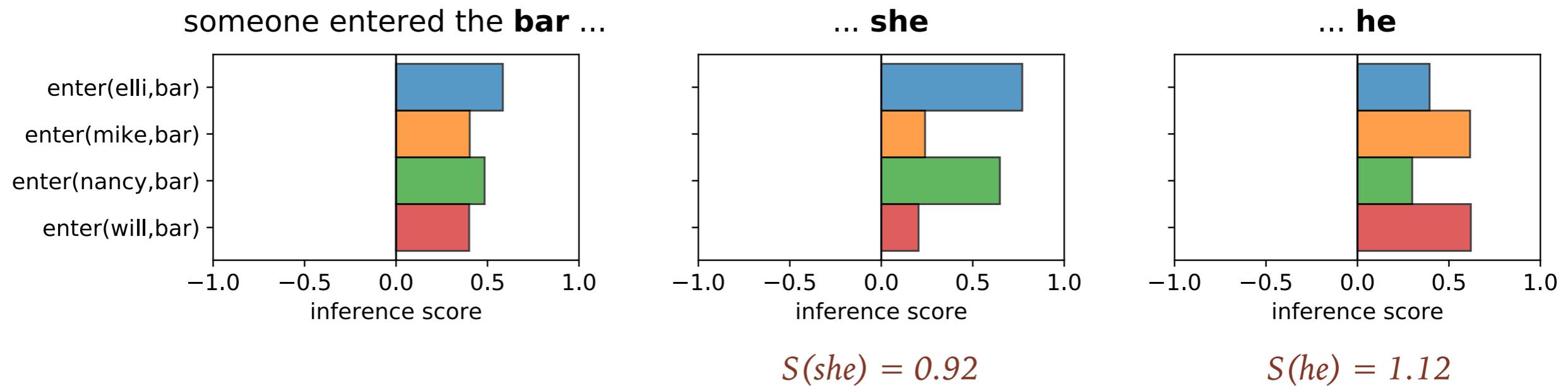
- Negation affects entailments and probabilistic inferences
- Interaction between negation and presupposition (triggered by “the”)
- Presupposition has an effect beyond the literal meaning



QUANTIFICATION AND REFERENCE

Quantified expressions induce inferential uncertainty

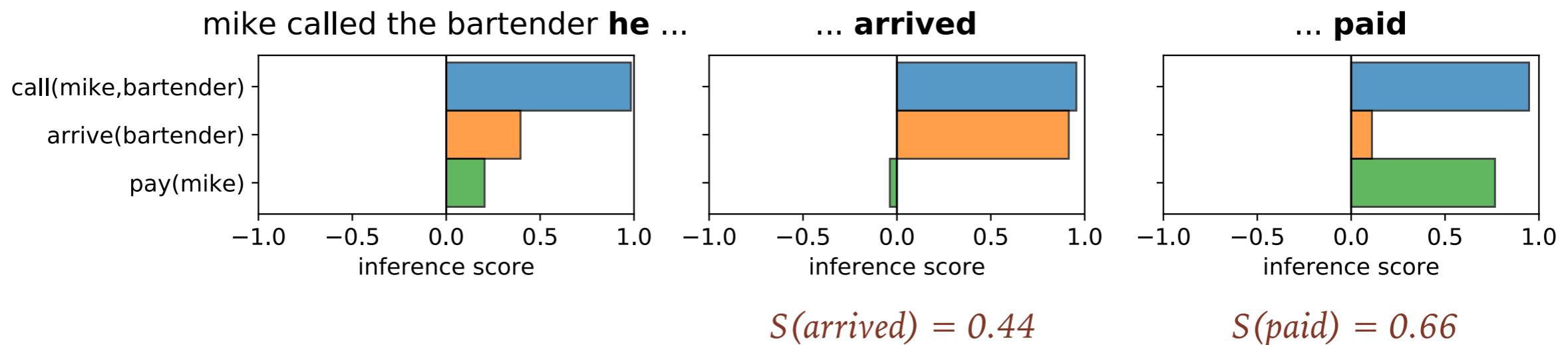
- Selective expressions (e.g. pronouns) can reduce this uncertainty
- Confirming initial expectations results in reduced **Surprisal**



REFERENTIAL AMBIGUITY

In the training data, the anaphoric antecedent of pronouns is always disambiguated by the preceding or the following context

- Ambiguous pronouns trigger competing hypotheses about the utterance-final interpretation
- Disambiguating continuations result in utterance-level entailments
- **Surprisal** estimates reflect difference between expected and unexpected continuations



SUMMARY

Distributional Formal Semantics

- Compositionality
- Entailment and probabilistic inference
- Incremental meaning construction

Distributional Semantics

- Semantic similarity
- Empirically driven
- Cognitively inspired

?

DS VS. DFS: COMPLEMENTARY ASPECTS OF MEANING

➤ Semantic similarity:

lexical similarity
beer ~ wine

vs.

propositional similarity
order(mike,beer) ~ drink(mike,beer)

➤ Data-driven sampling:

bottom-up

vs.

top-down

individual linguistic co-occurrences

high-level description of the world

➤ Cognitive foundation:

semantic memory

vs.

utterance interpretation

feature-based word meanings

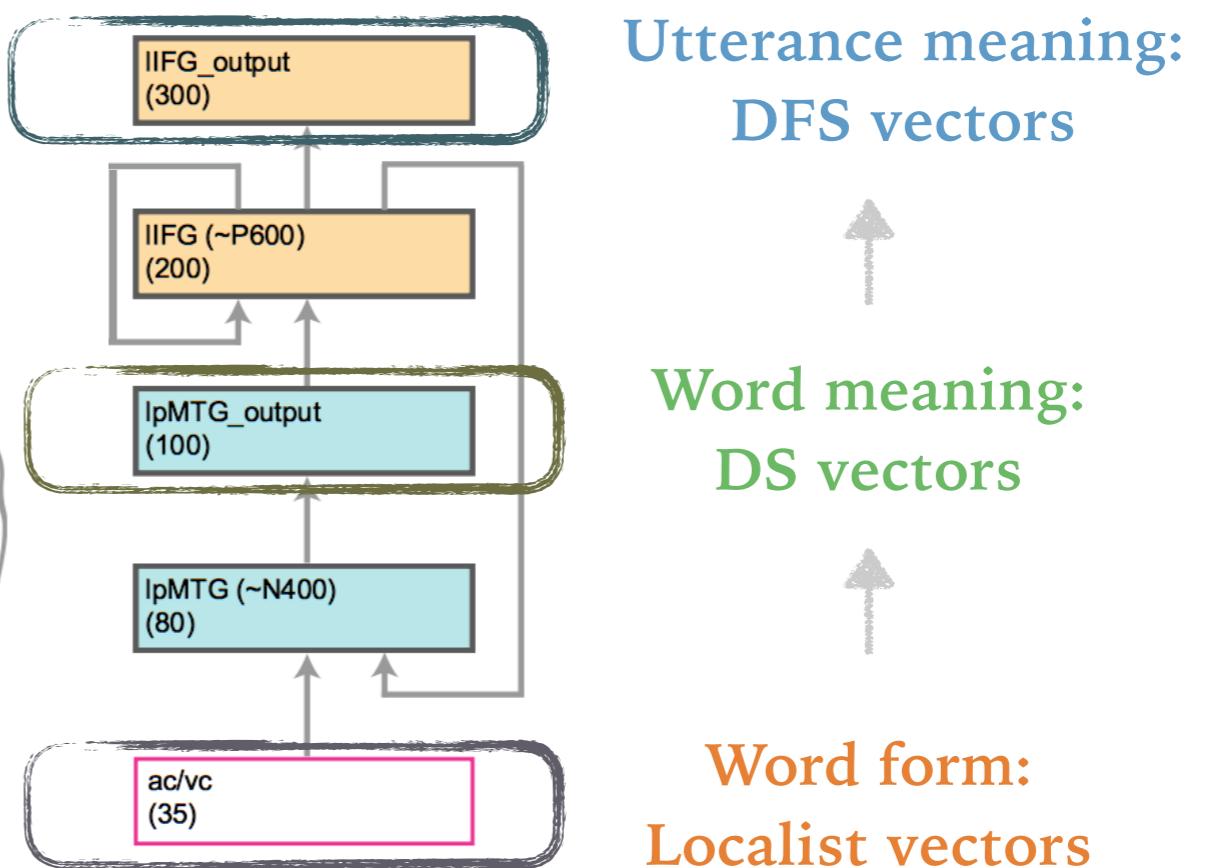
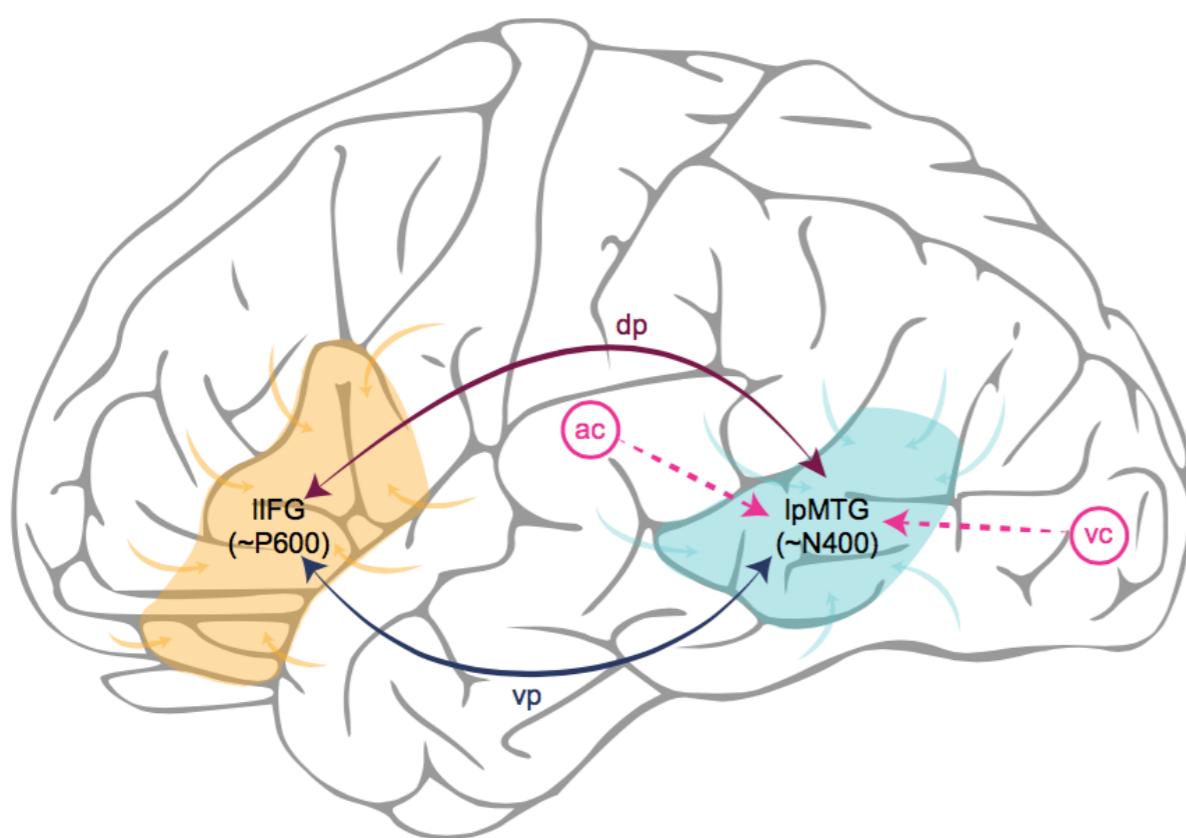
unfolding discourse-level interpretation

Kutas & Federmeier (2000); McRae et al. (2005); van Berkum (2009); Brouwer et al. (2012, 2017)

DS VS. DFS: A COGNITIVE MODEL OF LANGUAGE COMPREHENSION

The Retrieval-Integration account of the electrophysiology of language comprehension

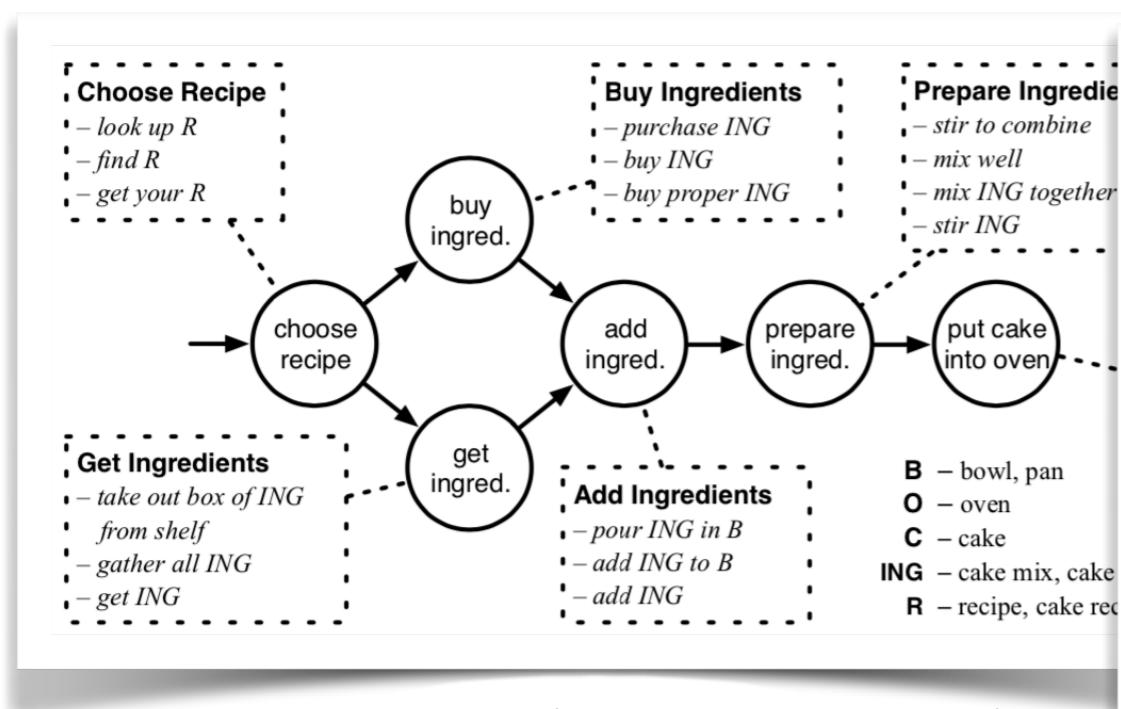
- Word meaning retrieval~N400
- Integration in utterance meaning~P600



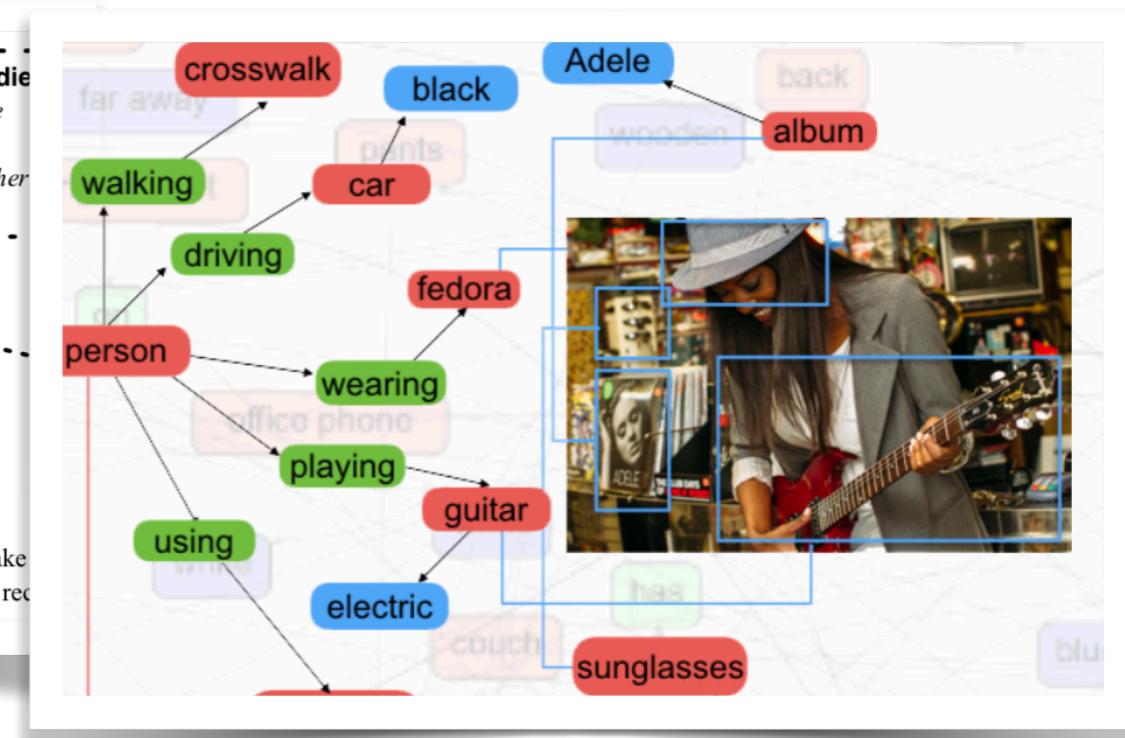
OUTLOOK: DATA-DRIVEN DFS

Goal: Employ DFS to model deep understanding using real-world propositional co-occurrences

- Step 1: Derive meaning space from resource with world knowledge about propositional (not: lexical) co-occurrence
- Step 2: Mapping natural language to vectors in the meaning space



DeScript corpus (Wanzare et al., 2016)



DISTRIBUTIONAL FORMAL SEMANTICS

```

● ○ ● ✘ 81
riwrap /Users/noortje/git/mesh/mesh model.mesh

[model:train> dssScores train "the waiter didnt bring cola"
Sentence: "the waiter didnt bring cola"
Semantics: "(!bring(waiter,cola) & referent(waiter))"

Sample: 99 / 100
the waiter didnt bring cola
+0.03976 +0.07265 +0.11242 +0.12072 +0.23314 +0.05547 +0.28862 +0.49729 +0.78590
mike entered the bar -0.03631 -0.06161 -0.09793 -0.04566 -0.14359 -0.02718 -0.17078 +0.04274 -0.12804 mike entered the bar
mike entered the restaurant +0.01022 +0.03858 +0.04880 -0.03375 +0.01505 -0.03782 -0.02277 +0.02725 +0.00447 mike entered the restaurant
mike entered a bar -0.03631 -0.06161 -0.09793 -0.04566 -0.14359 -0.02718 -0.17078 +0.04274 -0.12804 mike entered a bar
mike entered a restaurant +0.01022 +0.03858 +0.04880 -0.03375 +0.01505 -0.03782 -0.02277 +0.02725 +0.00447 mike entered a restaurant
mike called the bartender +0.03188 -0.08045 -0.04856 +0.05735 +0.00878 +0.02954 +0.03832 -0.14309 -0.10477 mike called the bartender
mike called the waiter +0.03599 +0.06212 +0.09811 +0.01996 +0.11807 -0.00443 +0.11364 +0.06383 +0.17747 mike called the waiter
mike ordered cola -0.01004 -0.04187 -0.05190 +0.00068 -0.05122 -0.05334 -0.10456 +0.03896 -0.30985 mike ordered cola
mike ordered water -0.01538 +0.04681 +0.03143 -0.08172 -0.05029 -0.00761 -0.05790 +0.06089 +0.00299 mike ordered water
mike ordered fries -0.03341 -0.01147 -0.04488 -0.04774 -0.09262 +0.03828 -0.05433 +0.07146 +0.01713 mike ordered fries
mike ordered salad -0.02789 -0.03612 -0.06400 -0.07984 -0.14384 -0.03404 -0.17789 +0.00563 -0.17226 will entered the bar
mike paid +0.02613 +0.03011 +0.05624 +0.00372 +0.05996 -0.00853 +0.05143 -0.04377 +0.00766 mike paid
will entered the bar +0.00349 +0.02943 +0.03292 -0.00720 +0.02571 +0.00934 +0.03505 -0.01658 +0.01847 will entered the restaurant
will entered a bar -0.02789 -0.03612 -0.06400 -0.07984 -0.14384 -0.03404 -0.17789 +0.00563 -0.17226 will entered a bar
will entered a restaurant +0.00349 +0.02943 +0.03292 -0.00720 +0.02571 +0.00934 +0.03505 -0.01658 +0.01847 will entered a restaurant
will called the bartender +0.02076 -0.13767 -0.11691 +0.06857 -0.04834 -0.03780 -0.08614 +0.00635 -0.07979 will called the bartender
will called the waiter +0.08083 +0.04298 +0.12380 +0.03795 +0.16176 +0.02265 +0.18441 -0.04373 +0.14068 will called the waiter
will ordered cola -0.00965 +0.00586 +0.01551 -0.00487 +0.01064 +0.00167 +0.01231 -0.35065 -0.33833 will ordered cola
will ordered water +0.01004 -0.05081 -0.04078 -0.05595 -0.09672 +0.04801 -0.04872 +0.01078 +0.05207 will ordered water
will ordered fries -0.02149 -0.01640 -0.03789 -0.01172 -0.04961 +0.05095 +0.00134 +0.05273 +0.05408 will ordered fries
will ordered salad -0.00830 -0.00519 +0.00311 -0.22862 -0.22550 -0.05572 -0.28122 +0.14156 -0.13966 will ordered salad
will paid +0.03429 -0.03742 -0.00313 -0.08126 -0.08439 +0.07986 -0.00453 -0.01062 +0.01515 will paid
elli entered the bar +0.02483 -0.02186 +0.00297 -0.11115 -0.10818 -0.03680 -0.14498 +0.03620 -0.10878 elli entered the bar
elli entered the restaurant +0.03567 +0.03244 -0.00323 +0.03621 +0.03298 +0.01720 +0.05018 +0.02189 +0.07206 elli entered the restaurant
elli entered a bar +0.02483 -0.02186 +0.00297 -0.11115 -0.10818 -0.03680 -0.14498 +0.03620 -0.10878 elli entered a bar
elli entered a restaurant -0.03567 +0.03244 -0.00323 +0.03621 +0.03298 +0.01720 +0.05018 +0.02189 +0.07206 elli entered a restaurant
elli called the bartender +0.06144 -0.03906 +0.02238 -0.01420 +0.00818 -0.02667 -0.01849 +0.06731 +0.04882 elli called the bartender
elli called the waiter +0.04167 +0.06759 +0.10926 -0.00536 +0.10390 +0.02676 +0.13066 -0.01374 +0.11691 elli called the waiter
elli ordered cola +0.01063 +0.00412 +0.01475 -0.02892 -0.01417 -0.05965 -0.07382 -0.31182 -0.38564 elli ordered cola
elli ordered water -0.05793 -0.07814 -0.13607 -0.13352 -0.26959 +0.02940 -0.24055 +0.05797 -0.18259 elli ordered water
elli ordered fries +0.01816 -0.00623 +0.01193 -0.00553 +0.00640 +0.00965 +0.01604 +0.01939 +0.03544 elli ordered fries
elli ordered salad -0.03949 -0.02396 -0.06345 +0.03649 -0.02697 -0.05009 -0.07705 +0.05115 -0.02591 elli ordered salad
elli paid +0.02955 +0.02126 +0.05082 -0.03215 +0.01867 -0.01120 +0.00746 +0.03732 +0.04479 elli paid
nancy entered the bar +0.00632 -0.01544 -0.00912 -0.04595 -0.05507 +0.01850 -0.03658 +0.03667 +0.00009 nancy entered the bar
nancy entered the restaurant +0.01554 +0.04885 +0.06440 -0.01300 +0.05139 -0.02250 +0.02889 +0.04373 +0.07262 nancy entered the restaurant
nancy entered a bar +0.00632 -0.01544 -0.00912 -0.04595 -0.05507 +0.01850 -0.03658 +0.03667 +0.00009 nancy entered a bar
nancy entered a restaurant +0.01554 +0.04885 +0.06440 -0.01300 +0.05139 -0.02250 +0.02889 +0.04373 +0.07262 nancy entered a restaurant
nancy called the bartender +0.02178 -0.10963 -0.08785 -0.04226 -0.13011 +0.04749 -0.08262 -0.00824 -0.09085 nancy called the bartender
nancy called the waiter +0.04144 +0.06182 +0.10327 -0.02039 +0.08288 -0.01541 +0.06746 -0.01932 +0.04814 nancy called the waiter
nancy ordered cola +0.00239 -0.00879 -0.00640 -0.04315 -0.04955 +0.06494 +0.01539 -0.28420 -0.26880 nancy ordered cola
nancy ordered water -0.03059 -0.04221 -0.01162 -0.08900 -0.07738 -0.04195 -0.11933 +0.04696 -0.07237 nancy ordered water
nancy ordered fries +0.02833 +0.01130 +0.03963 -0.06632 -0.02669 -0.03772 -0.06441 +0.00215 -0.06226 nancy ordered fries
nancy ordered salad +0.00221 +0.00496 +0.00717 +0.02244 +0.02961 -0.01121 +0.01840 +0.04451 +0.06291 nancy ordered salad
nancy paid +0.04789 +0.03190 +0.07978 -0.06158 +0.01820 -0.00296 +0.01524 -0.06668 -0.05145 nancy paid
the bartender arrived +0.05815 -0.13320 -0.07505 +0.00261 -0.07244 +0.05968 -0.01276 -0.01742 -0.03018 the bartender arrived
the bartender brought cola +0.07821 -0.10183 -0.02362 +0.00978 -0.01384 +0.02501 +0.01117 -0.33560 -0.32443 the bartender brought cola
the bartender brought water +0.04493 -0.14682 -0.10189 -0.00642 -0.10831 +0.01195 -0.09636 +0.03999 -0.05637 the bartender brought water
the bartender brought fries +0.08262 -0.11857 -0.03595 -0.09690 -0.13286 -0.02558 -0.15844 +0.01793 -0.14044 the bartender brought fries

```



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mentary
er, given the
pproaches—
has proven
which
of formal
entations
ntal semantic
how the

probabilistic inference, and
how the information-theoretic notion of ‘surprisal’ (measured in Entropy and

<https://github.com/hbrouwer/mesh>

Surprisal) naturally follows representations can be derived incrementally from linguistic input using a recurrent neural network model, and how the resultant incremental semantic construction procedure intuitively captures key semantic phenomena, including negation, presupposition, and anaphoricity.

DFS-TOOLS

<https://github.com/hbrouwer/dfs-tools>