DISTRIBUTIONAL FORMAL SEMANTICS

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NATURAL LANGUAGE SEMANTICS

Model-theoretic Semantics

- Truth-conditional meaning
- Logical entailment
- Compositionality

?

Distributional Semantics

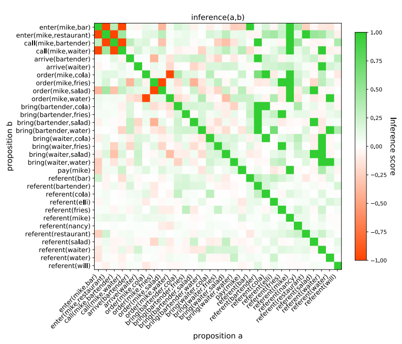
- Semantic similarity
- Empirically driven
- Cognitively inspired

E.g., Baroni et al. (2010,2014); Boleda & Herbelot (2016); Coecke et al. (2010); Grefenstette & Sadrzadeh (2011); Socher et al. (2012)

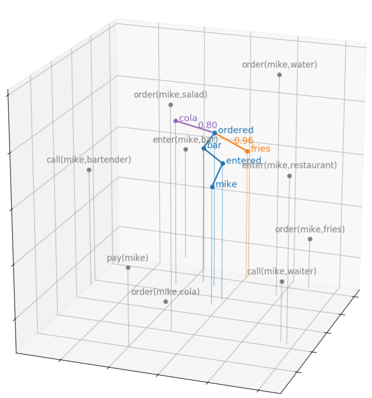
A FRAMEWORK FOR DISTRIBUTIONAL FORMAL SEMANTICS

	p_1	p_2	p_3	$\cdots$
M_1	1	1	0	…
M_2	1	0	0	…
M_3	0	1	0	…
M_4	1	1	1	…
…	…	…	…	…

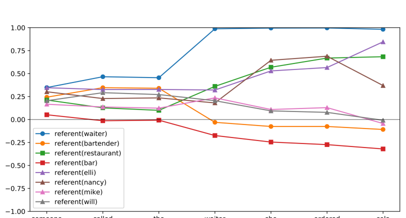
A meaning space for Distributional Formal Semantics



Formal properties of the meaning space



Incremental meaning construction



Semantic processing in the meaning space

FROM MODELS TO MEANING SPACE

$M_1 = \langle U_1, V_1 \rangle$

$p_1 \wedge \neg p_2 \wedge p_3 \wedge \ldots$

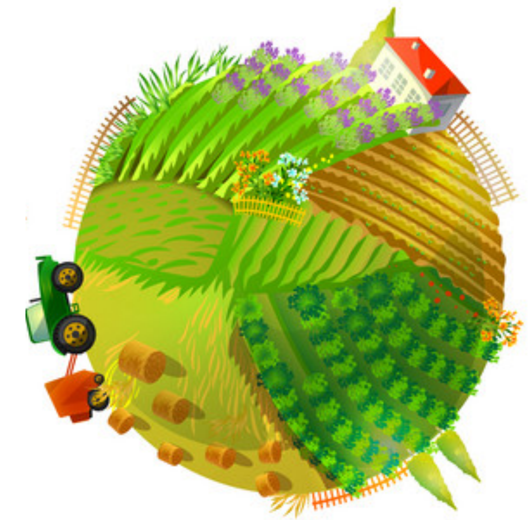
$M_2 = \langle U_2, V_2 \rangle$

$\neg p_1 \wedge p_2 \wedge p_3 \wedge \ldots$

$M_3 = \langle U_3, V_3 \rangle$

$\neg p_1 \wedge p_2 \wedge \neg p_3 \wedge \ldots$

…



$M_n = \langle U_n, V_n \rangle$

$\neg p_1 \wedge \neg p_2 \wedge \neg p_3 \wedge \ldots$

- The set of models $\mathcal{M}_\mathcal{P}$ — describing states-of-affairs over propositions in $\mathcal{P}$ — defines a meaning space
- Propositional meaning defined by co-occurrence across models

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2021, Inf. Comp.)

CAPTURING THE STRUCTURE OF THE WORLD

"A boy rides a bike"

Boy is (likely) outside

Boy is not asleep

If it's evening, the light is on

The bike has wheels

etc.

World knowledge restricts propositional co-occurrence in the meaning space derived from the set of models $\mathcal{M}_\mathcal{P}$

- **Hard** world knowledge constraints restrict individual models
- **Probabilistic** constraints define probabilistic co-occurrences across the set of models $\mathcal{M}_\mathcal{P}$

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2021, Inf. Comp.)

DFS MEANING SPACE $S_{\mathcal{M}\times\mathcal{P}}$

propositional meaning vectors

formal models

$$
\begin{array}{c|ccccc}
 & p_1 & p_2 & p_3 & p_4 & \dots \\
\hline
M_1 & 1 & 1 & 0 & 0 & \dots \\
M_2 & 1 & 0 & 0 & 1 & \dots \\
M_3 & 0 & 1 & 0 & 1 & \dots \\
M_4 & 1 & 1 & 1 & 1 & \dots \\
M_5 & 0 & 1 & 0 & 0 & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots
\end{array}
$$

$$[\![p_j]\!]^{\mathcal{M}} := v(p_j)$$

where: $v_i(p_j) = 1$ *iff* $M_i \vDash p_j$

- **Incremental inference-based probabilistic sampling:** Based on a set of propositions $\mathcal{P}$, we sample a set of models $\mathcal{M}_{\mathcal{P}}$—taking into account hard and probabilistic world knowledge constraints
- **Co-occurrence defines meaning:** Propositions with related meanings are true in many of the same models, resulting in similar meaning vectors

Venhuizen, Hendriks, Crocker, Brouwer (2019, WoLLIC; 2021, Inf. Comp.)

THE DISTRIBUTIONAL HYPOTHESIS REVISITED

"

You shall know a ~~word~~ *proposition*
by the company it keeps

- J. R. Firth (1957)

MEANING VECTOR COMPOSITION

Meaning vectors can be combined to define compositional meanings

- Standard logical operators interpreted as in model-theory

  $v_i(\neg p) \quad = 1 \quad$ *iff* $\; M_i \nvDash p$

  $v_i(p \wedge q) \quad = 1 \quad$ *iff* $\; M_i \vDash p$ *and* $M_i \vDash q$

  … *etc.*

- Quantification is defined relative to the combined universe of $\mathcal{M}_\mathcal{P}$: $\mathcal{U}_\mathcal{M} = \{e_{1 \ldots} e_m\}$ (thereby preserving entailment in $\mathcal{M}_\mathcal{P}$)

  $v_i(\forall x \varphi) \quad = 1 \quad$ *iff* $\; M_i \vDash \varphi[x \backslash e_1] \wedge \ldots \wedge \varphi[x \backslash e_m]$

  $v_i(\exists x \varphi) \quad = 1 \quad$ *iff* $\; M_i \vDash \varphi[x \backslash e_1] \vee \ldots \vee \varphi[x \backslash e_m]$

Venhuizen, Hendriks, Crocker, Brouwer (2021, Inf. Comp.)

PROBABILITIES IN THE MEANING SPACE

All (sub-)propositional meaning vectors inherently encode (co-)occurrence probabilities

- Prior probability of meaning vector *a*

$$P(a) = \frac{1}{|\mathcal{M}|} \sum_i \vec{v}_i(a)$$

- Conjunction probability between *a* and *b*

$$P(a \wedge b) = \frac{1}{|\mathcal{M}|} \sum_i \vec{v}_i(a) \vec{v}_i(b)$$

- Conditional probability of *a* given *b*

$$P(a|b) = \frac{P(a \wedge b)}{P(b)}$$

	p_1	p_2	p_3	p_4	
M_1	1	1	0	0	…
M_2	1	0	0	1	…
M_3	0	1	0	1	…
M_4	1	1	1	1	…
	0	1	0	0	…
	…	…	…	…	…

Venhuizen, Hendriks, Crocker, Brouwer (2021, Inf. Comp.)

QUANTIFYING PROBABILISTIC INFERENCE

Probabilistic logical inference of meaning vector *a* given *b*

$$inference(a,b) = \begin{cases} [P(a|b) - P(a)] \,/\, [1 - P(a)] & \text{if } P(a|b) > P(a) \\ [P(a|b) - P(a)] \,/\, P(a) & \text{otherwise} \end{cases}$$

- $P(a|b) > P(a)$: Positive inference (b increases probability of a)

$$inference(a,b) = 1 \Leftrightarrow b \vDash a$$

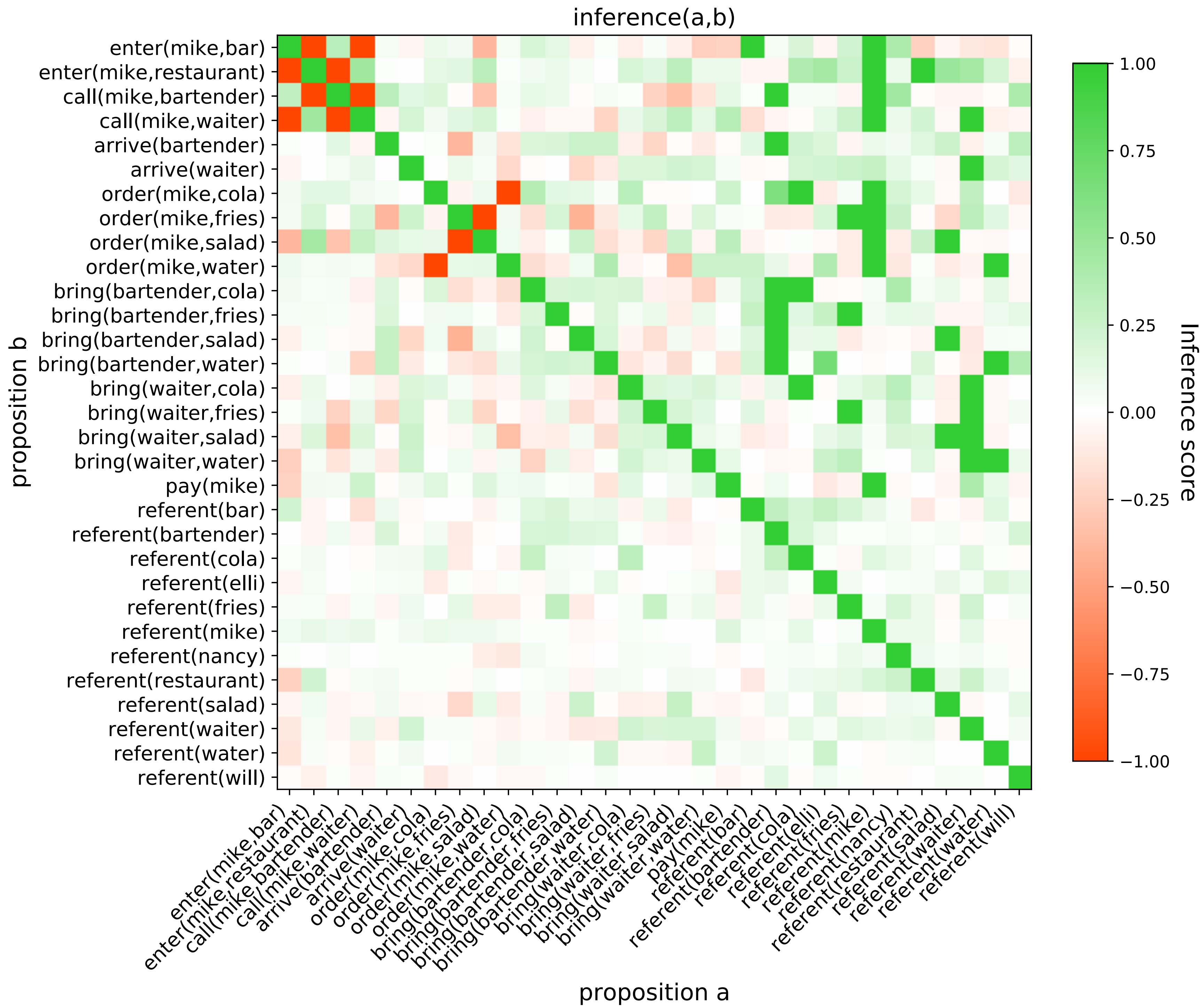
- $P(a|b) \leq P(a)$: Negative inference (b decreases probability of a)

$$inference(a,b) = -1 \Leftrightarrow b \vDash \neg a$$

Frank et al. (2009,Cognition)

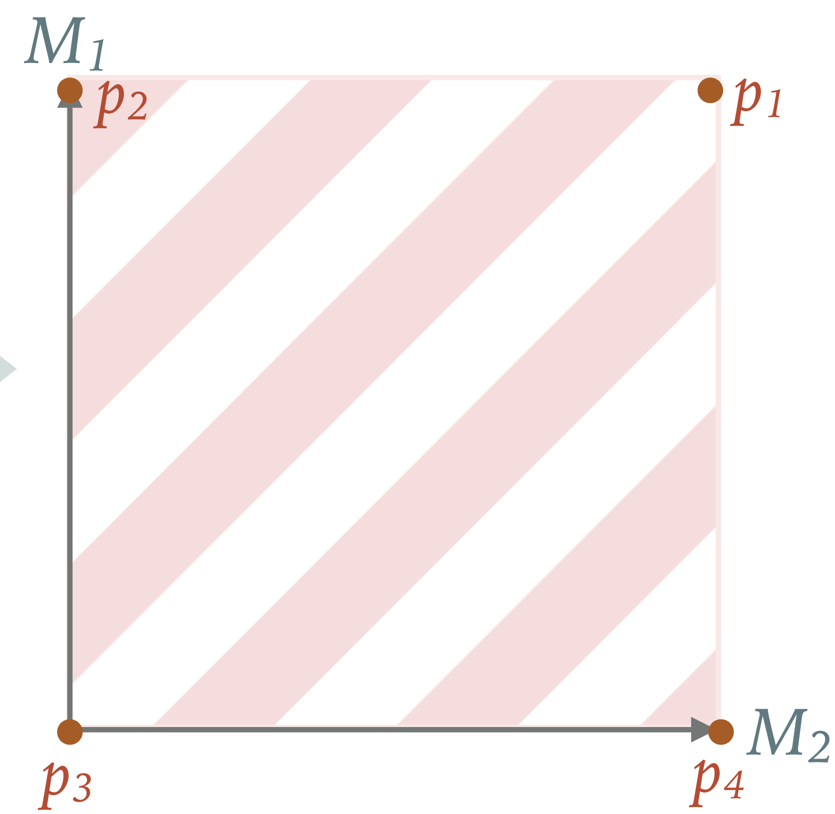
WORLD KNOWLEDGE IN THE MEANING SPACE

We sampled a meaning space of 150 models describing 51 propositions



CONTINUOUS NATURE OF THE MEANING SPACE

	p_1	p_2	p_3	p_4	
M_1	1	1	0	0	…
M_2	1	0	0	1	…
M_3	0	1	0	1	…
M_4	1	1	1	1	…
M_5	0	1	0	0	…
	…	…	…	…	…

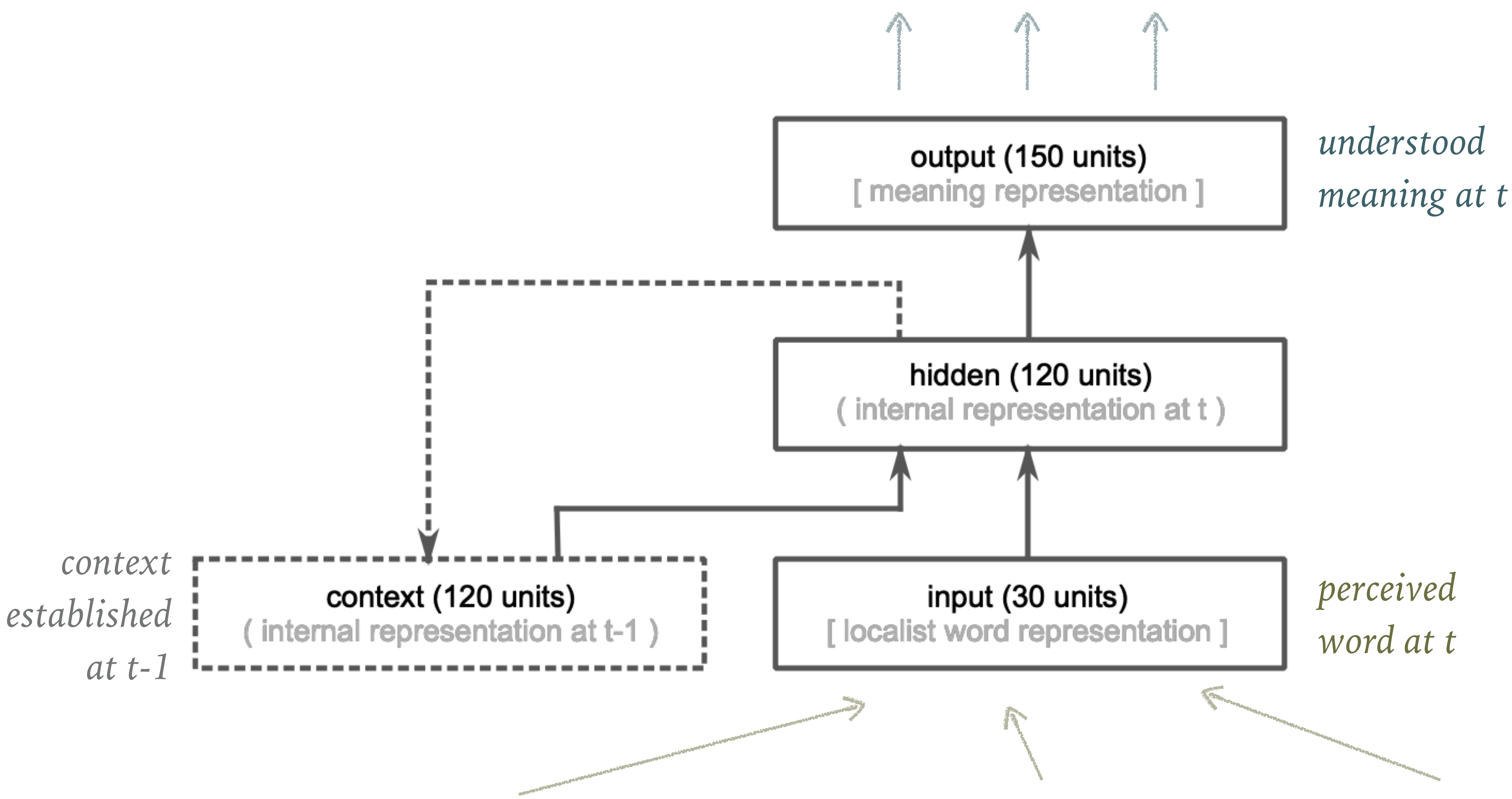


- Each point in the meaning space can be interpreted relative to $\mathcal{M}_{\mathcal{P}}$
  - Binary vectors: propositional meanings (simple or complex)
  - Real-valued vectors: sub-propositional meanings
- Sub-propositional meaning derives from incremental mapping from (sequences of) words to proposition-level meanings (Frank et al., 2009, *Cognition*)

A MODEL OF INCREMENTAL MEANING CONSTRUCTION

enter(mike,restaurant) ∧ order(mike,salad)

[1,0,1,0,0,1,1,0,1,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,...]



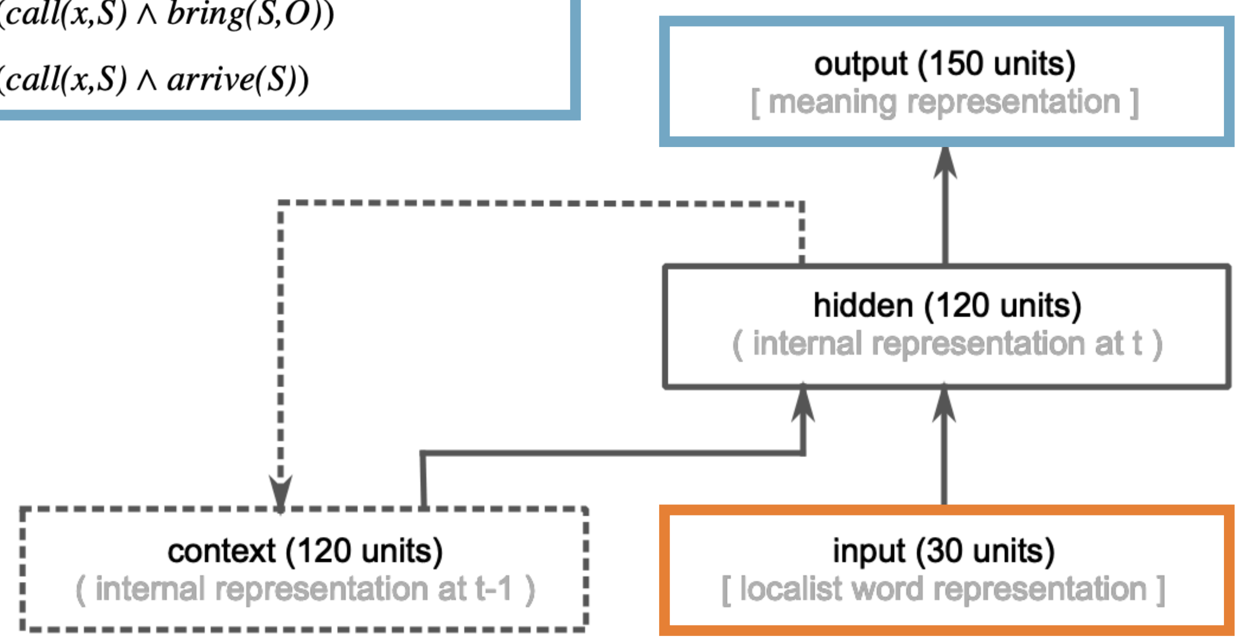
< [0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0], [0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0], [...] >

<"mike", "entered", "the", "restaurant", "and" "ordered", "salad">

"Mike entered the restaurant and ordered salad"

CONSTRUCTING THE MODEL: LANGUAGE

Utterance$_{[a/b]}$	Semantics$_a$	Semantics$_b$
P [entered/didn't enter] a *L*	$enter(P,L)$	$\neg(enter(P,L)) \wedge referent(P)$
P [entered/didn't enter] the *L*	$enter(P,L)$	$\neg(enter(P,L)) \wedge referent(P) \wedge referent(L)$
P [called/didn't call] the *S*	$call(P,S)$	$\neg(call(P,S)) \wedge referent(P) \wedge referent(S)$
P [ordered/didn't order] *O*	$order(P,O)$	$\neg(order(P,O)) \wedge referent(P)$
P [paid/didn't pay]	$pay(P)$	$\neg(pay(P)) \wedge referent(P)$
the *S* [arrived/didn't arrive]	$arrive(S)$	$\neg(arrive(S)) \wedge referent(S)$
the *S* [brought/didn't bring] *O*	$bring(S,O)$	$\neg(bring(S,O)) \wedge referent(S)$
[*P*/someone] entered the *L* he/she ordered *O*	$enter(P,L) \wedge order(P,O)$	$\exists x_{m/f}(enter(x,L) \wedge order(x,O))$
[*P*/someone] entered the *L* he/she called the *S*	$enter(P,L) \wedge call(P,S)$	$\exists x_{m/f}(enter(x,L) \wedge call(x,S))$
[*P*/someone] called the *S* he/she ordered *O*	$call(P,S) \wedge order(P,O)$	$\exists x_{m/f}(call(x,S) \wedge order(x,O))$
[*P*/someone] called the *S* he/she paid	$call(P,S) \wedge pay(P)$	$\exists x_{m/f}(call(x,S) \wedge pay(x))$
[*P*/someone] called the *S* he brought *O*	$call(P,S) \wedge bring(S,O)$	$\exists x(call(x,S) \wedge bring(S,O))$
[*P*/someone] called the *S* he arrived	$call(P,S) \wedge arrive(S)$	$\exists x(call(x,S) \wedge arrive(S))$

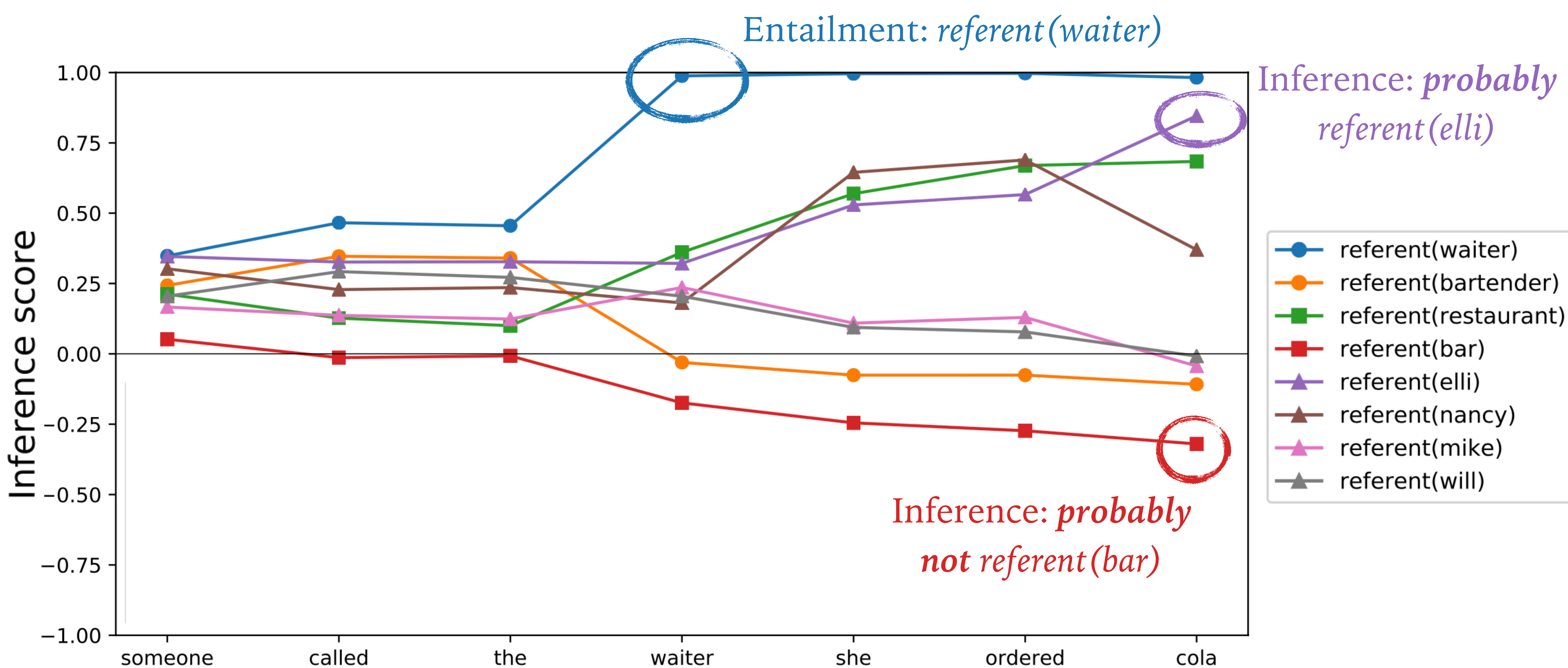


Venhuizen, Hendriks, Crocker, Brouwer (2021, Inf. Comp.)

ENTAILMENT AND INFERENCE

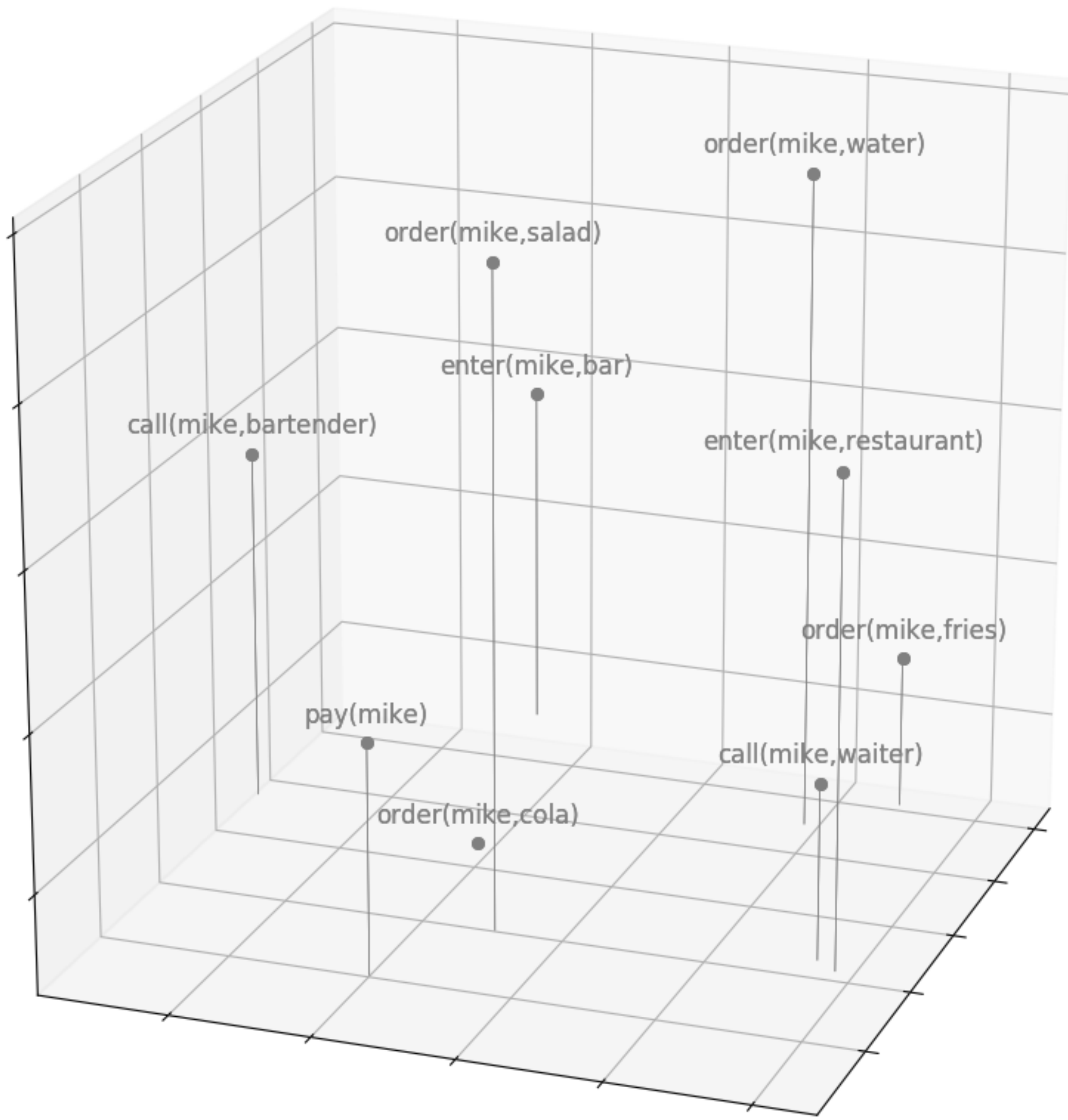
Incremental meaning construction in the model is driven by:

- Sentence-semantics mappings (literal utterance meaning)
- Structure of the meaning space (probabilistic inferences)



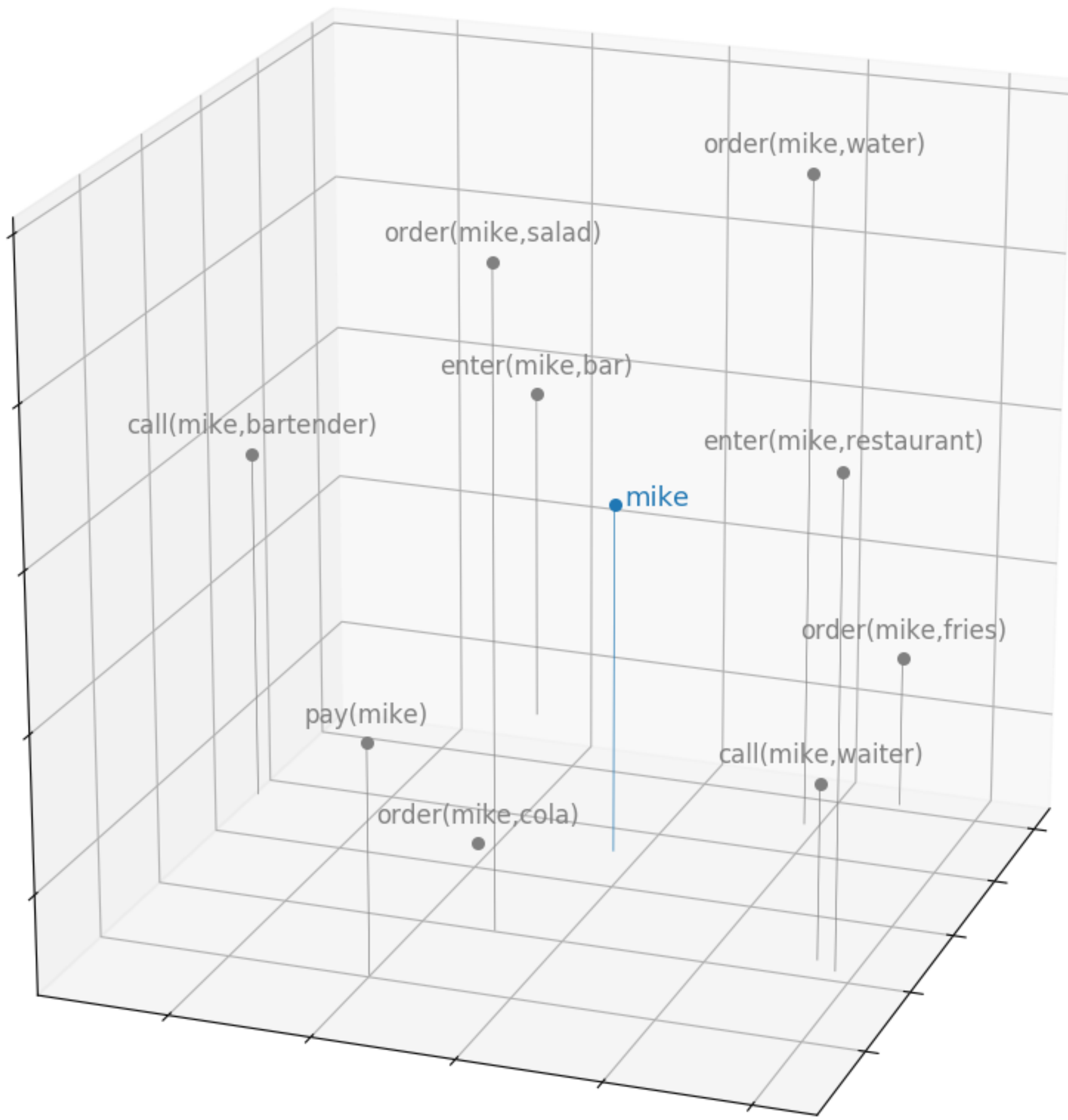
Venhuizen, Hendriks, Crocker, Brouwer (2021, Inf. Comp.)

MEANING SPACE NAVIGATION



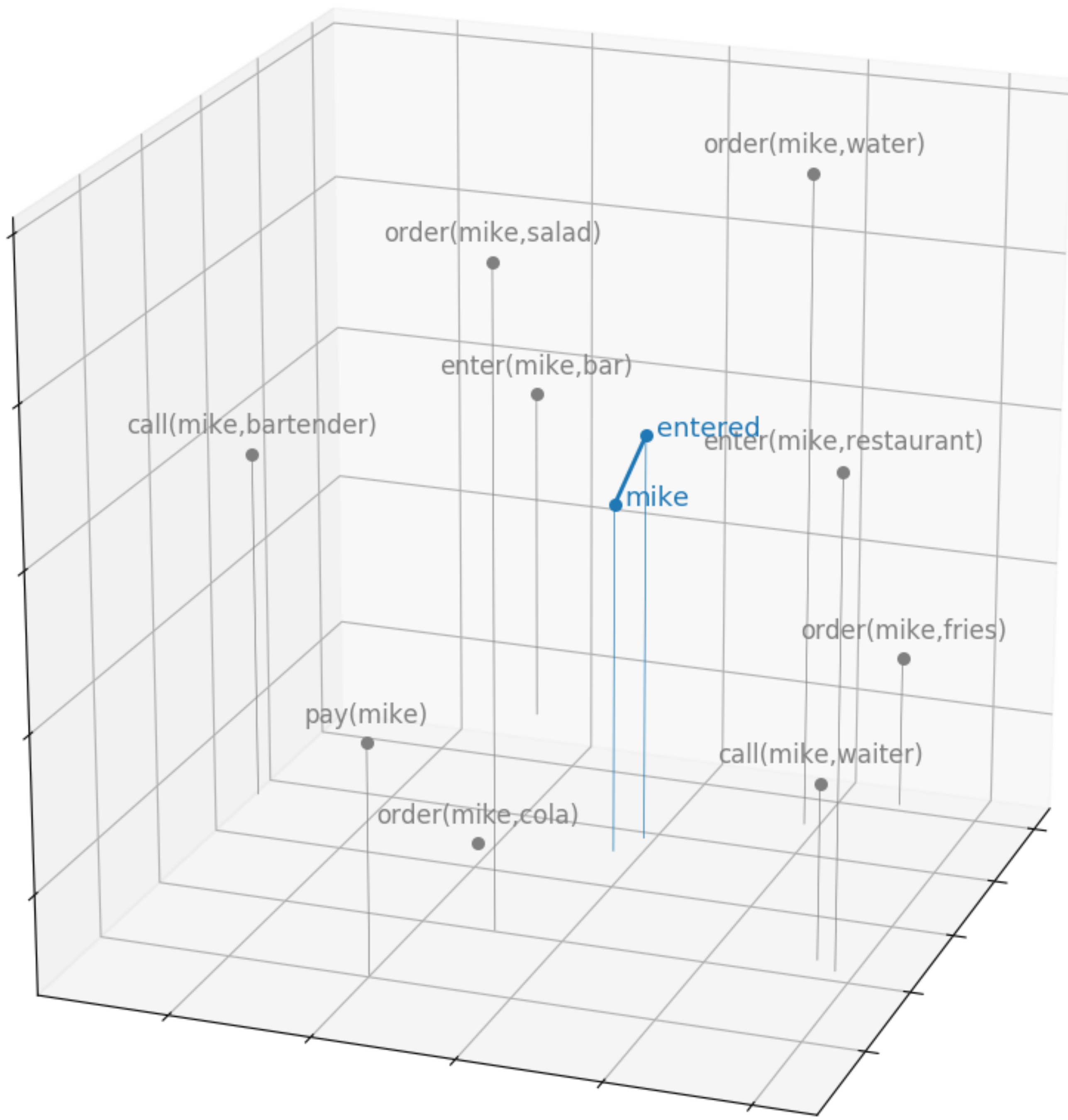
- ➤ Multi-dimensional scaling from 150 ➔ 3 dimensions
  (for a subset of the propositions)

MEANING SPACE NAVIGATION



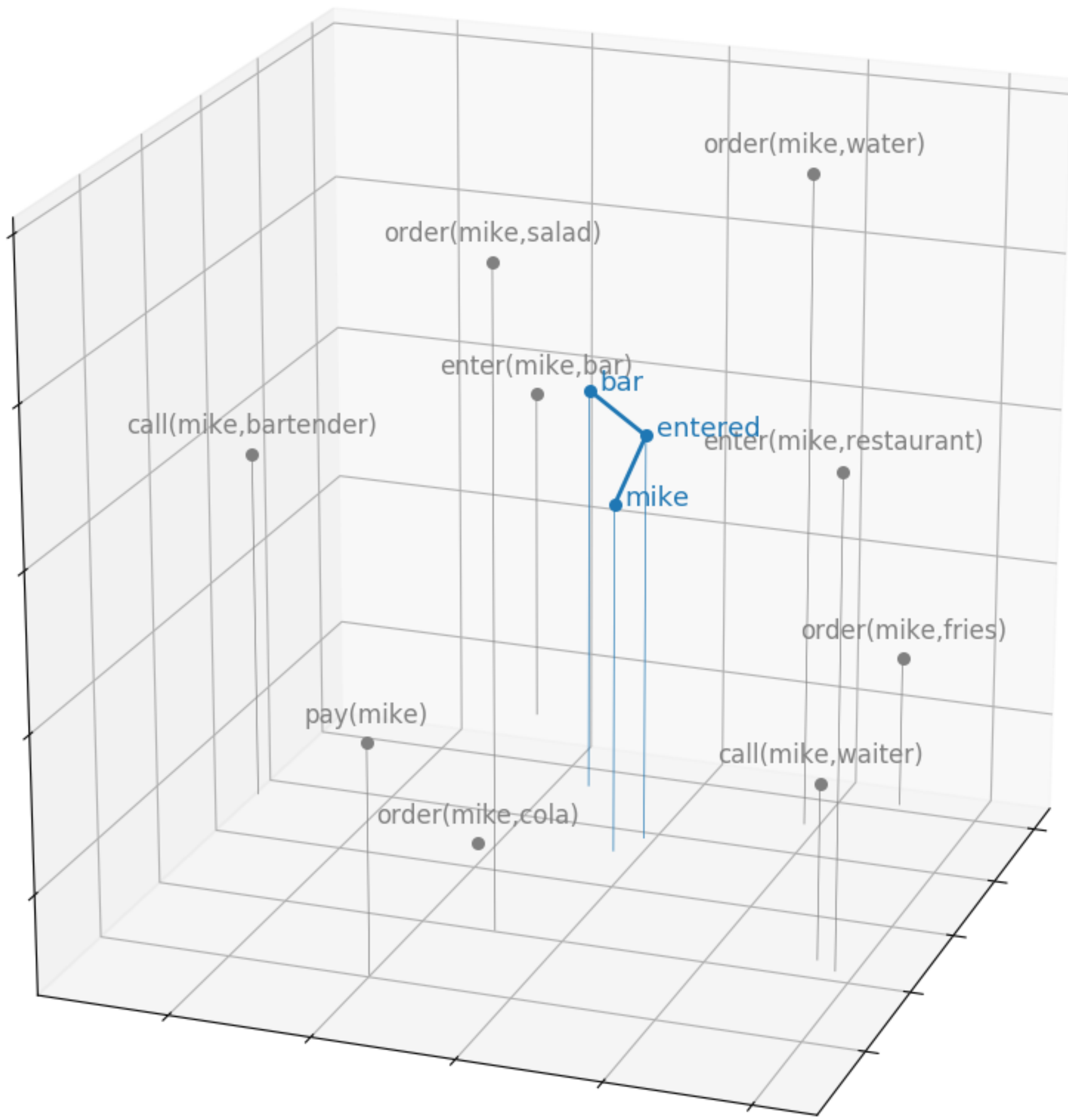
- Model-derived meaning of ‘*mike*’

MEANING SPACE NAVIGATION



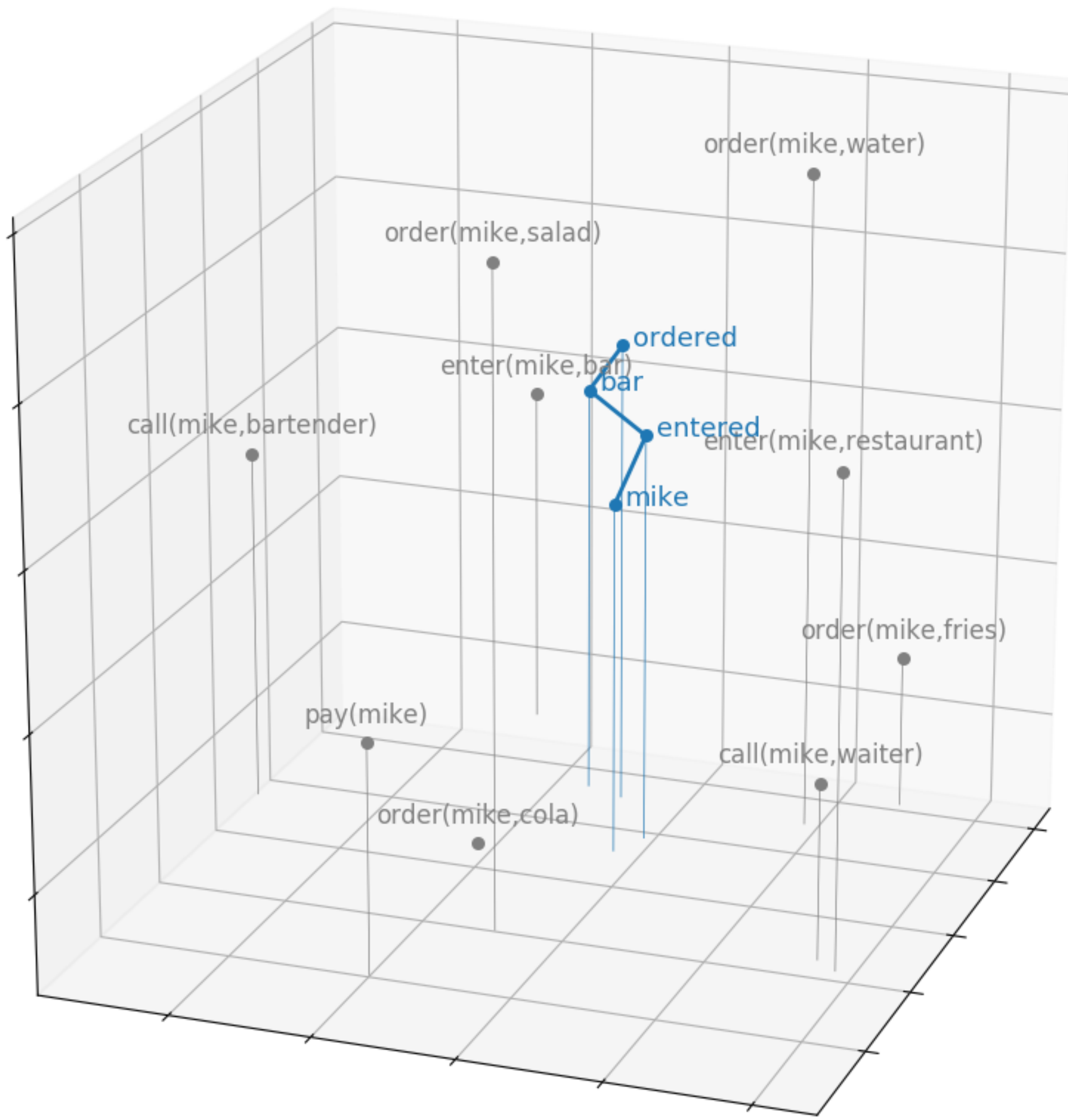
- Model-derived meaning of *'mike entered'*

MEANING SPACE NAVIGATION



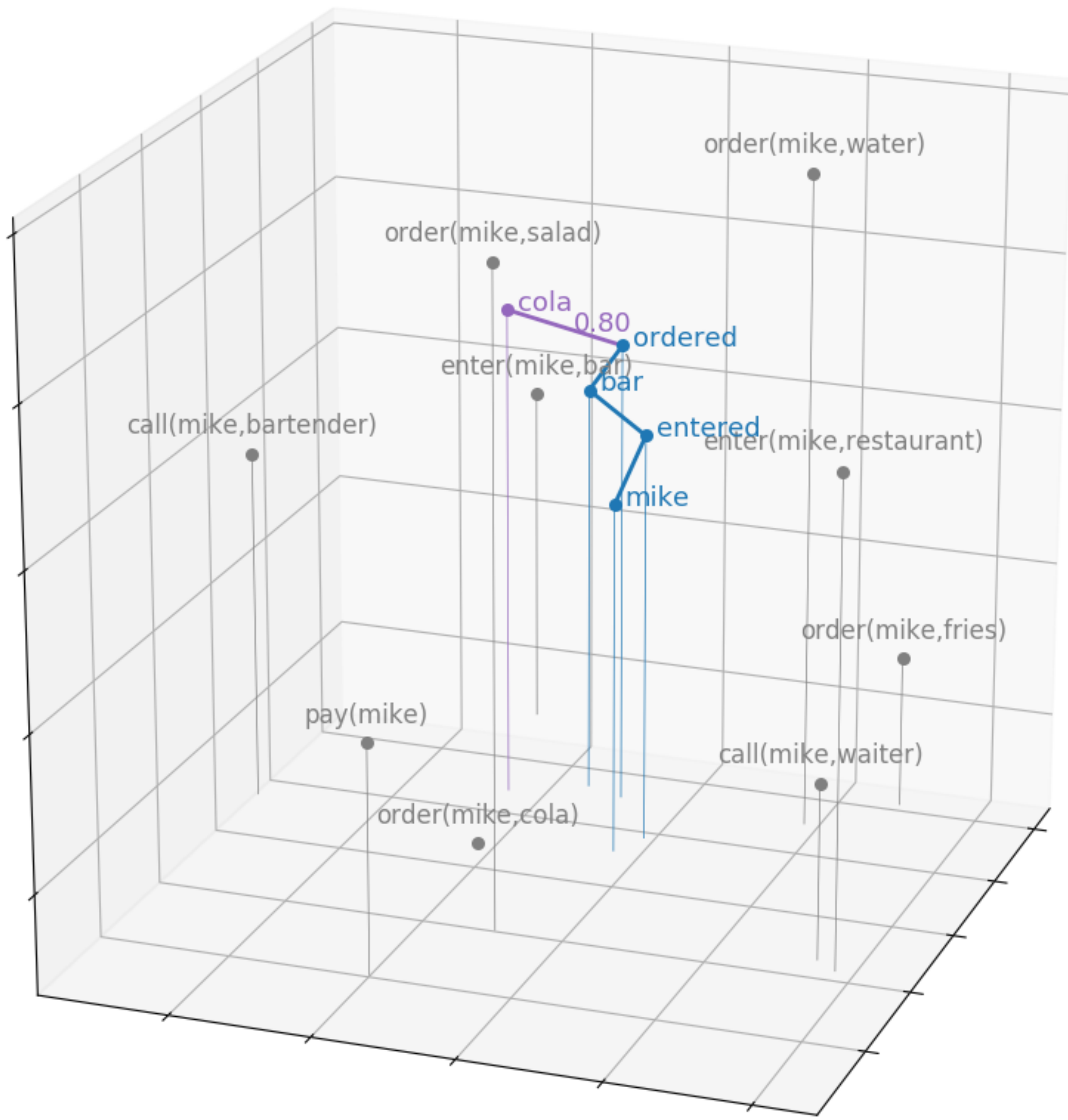
➤ Model-derived meaning of ‘*mike entered the bar*’

MEANING SPACE NAVIGATION



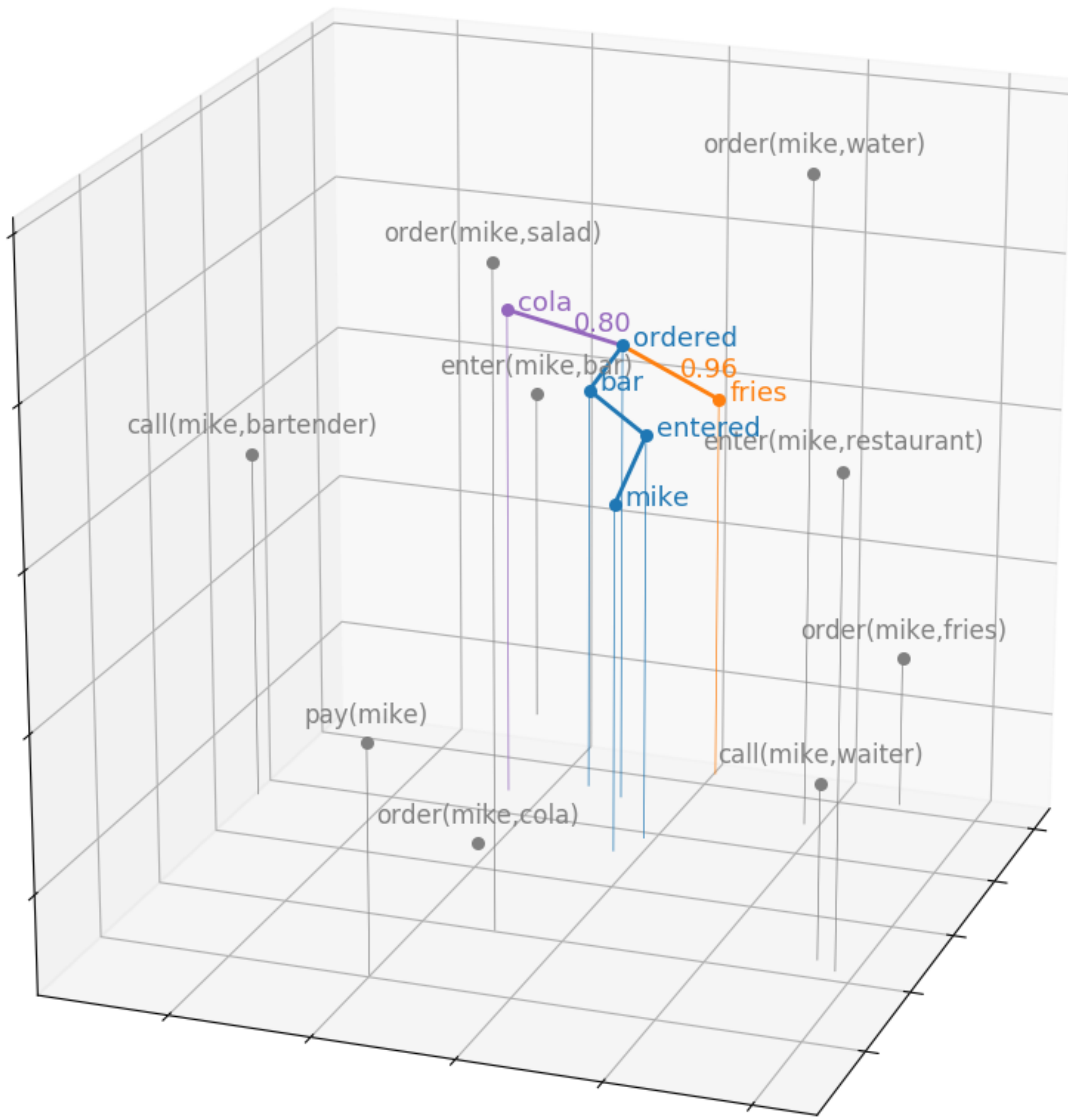
- Model-derived meaning of '*mike entered the bar [.] he ordered*'

MEANING SPACE NAVIGATION



- Model-derived meaning of ‘*mike entered the bar [.] he ordered cola*’
- Distance ∝ Surprisal: Expected transition in meaning space

MEANING SPACE NAVIGATION



- ➤ Model-derived meaning of ‘*mike entered the bar [.] he ordered fries*’
- ➤ Distance ∝ Surprisal: Unexpected transition in meaning space

INFORMATION THEORY IN DFS

Probabilistic nature of meaning space allows for defining formal notion of information (Shannon, 1948)

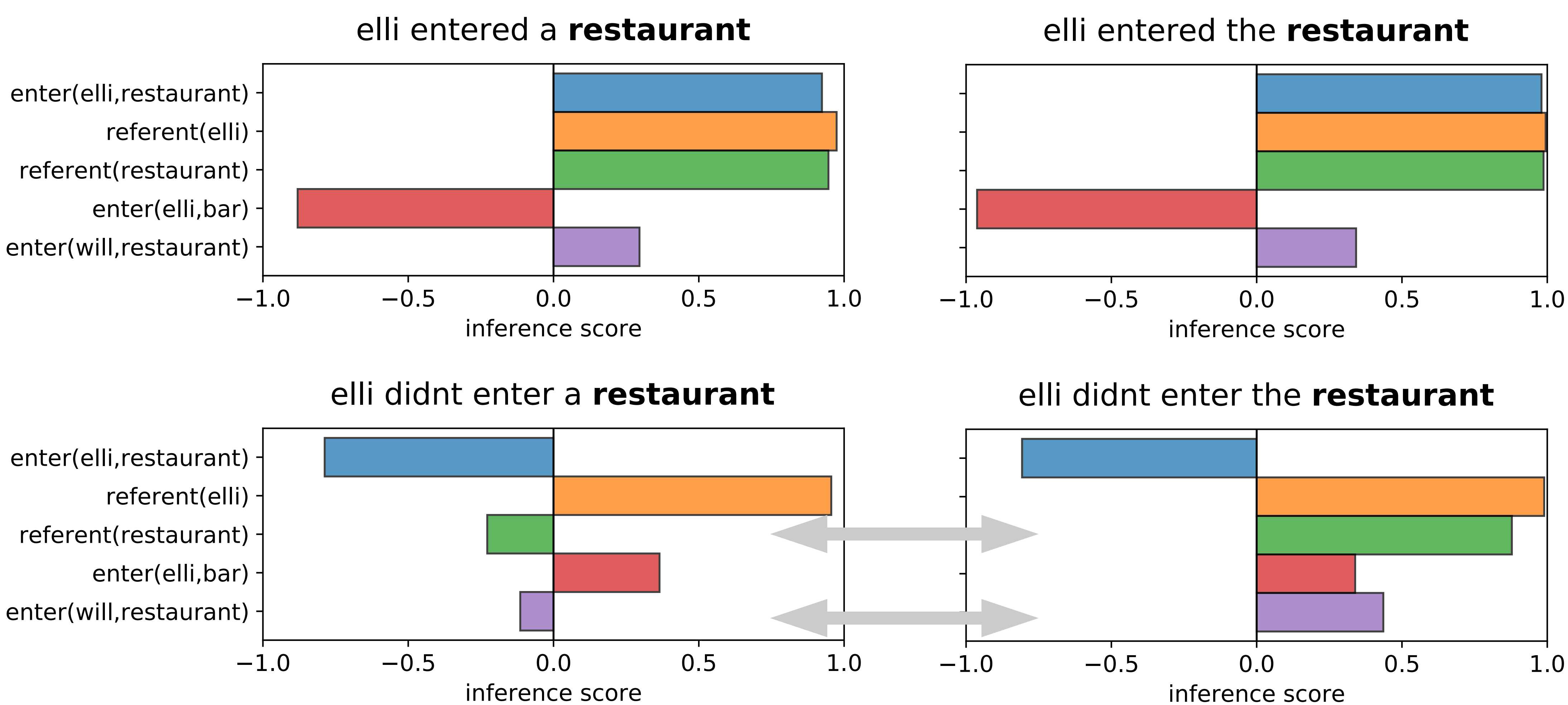
- **Surprisal** quantifies the expectancy of words in context
- Higher Surprisal ⇔ increased processing cost (Hale, 2001; Levy, 2008)
- In DFS, Surprisal quantifies expectancy of transition in meaning space, triggered by message m_{ab}:

$$S(m_{ab}) = -\log P(b|a)$$

➡ Word-by-word information effects of semantic construction

NEGATION AND PRESUPPOSITION

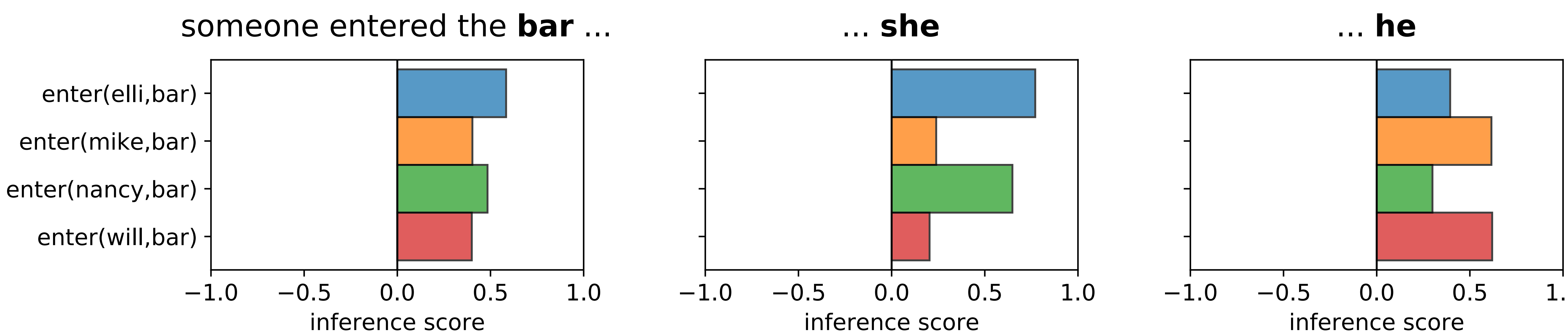
- Negation affects entailments and probabilistic inferences
- Interaction between negation and presupposition (triggered by *"the"*)
    - Presupposition has an effect beyond the literal meaning



QUANTIFICATION AND REFERENCE

Quantified expressions induce inferential uncertainty

- ➤ Selective expressions (e.g. pronouns) can reduce this uncertainty
- ➤ Confirming initial expectations results in reduced Surprisal



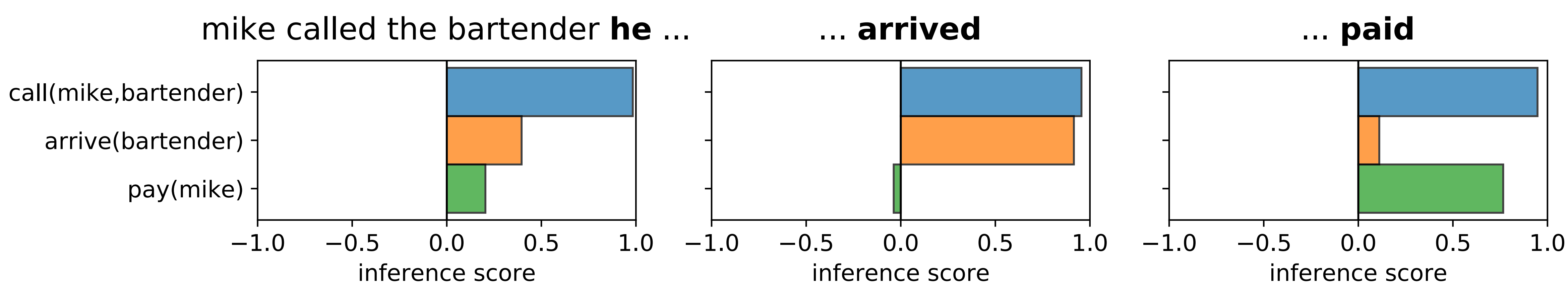
$S(she) = 0.92$ $S(he) = 1.12$

Venhuizen, Hendriks, Crocker, Brouwer (2021, Inf. Comp./ArXiv)

REFERENTIAL AMBIGUITY

In the training data, the anaphoric antecedent of pronouns is always disambiguated by the preceding or the following context

- Ambiguous pronouns trigger competing hypotheses about the utterance-final interpretation
- Disambiguating continuations result in utterance-level entailments
- Surprisal estimates reflect difference between expected and unexpected continuations



$S(arrived) = 0.44$ $S(paid) = 0.66$

Venhuizen, Hendriks, Crocker, Brouwer (2021, Inf. Comp./ArXiv)

SUMMARY

Distributional Formal Semantics

- Compositionality
- Entailment and probabilistic inference
- Incremental meaning construction

?

Distributional Semantics

- Semantic similarity
- Empirically driven
- Cognitively inspired

DS VS. DFS: COMPLEMENTARY ASPECTS OF MEANING

- **Semantic similarity:**

lexical similarity	vs.	propositional similarity
beer ∼ wine		*order(mike,beer) ∼ drink(mike,beer)*

- **Data-driven sampling:**

bottom-up	vs.	top-down
individual linguistic co-occurrences		*high-level description of the world*

- **Cognitive foundation:**

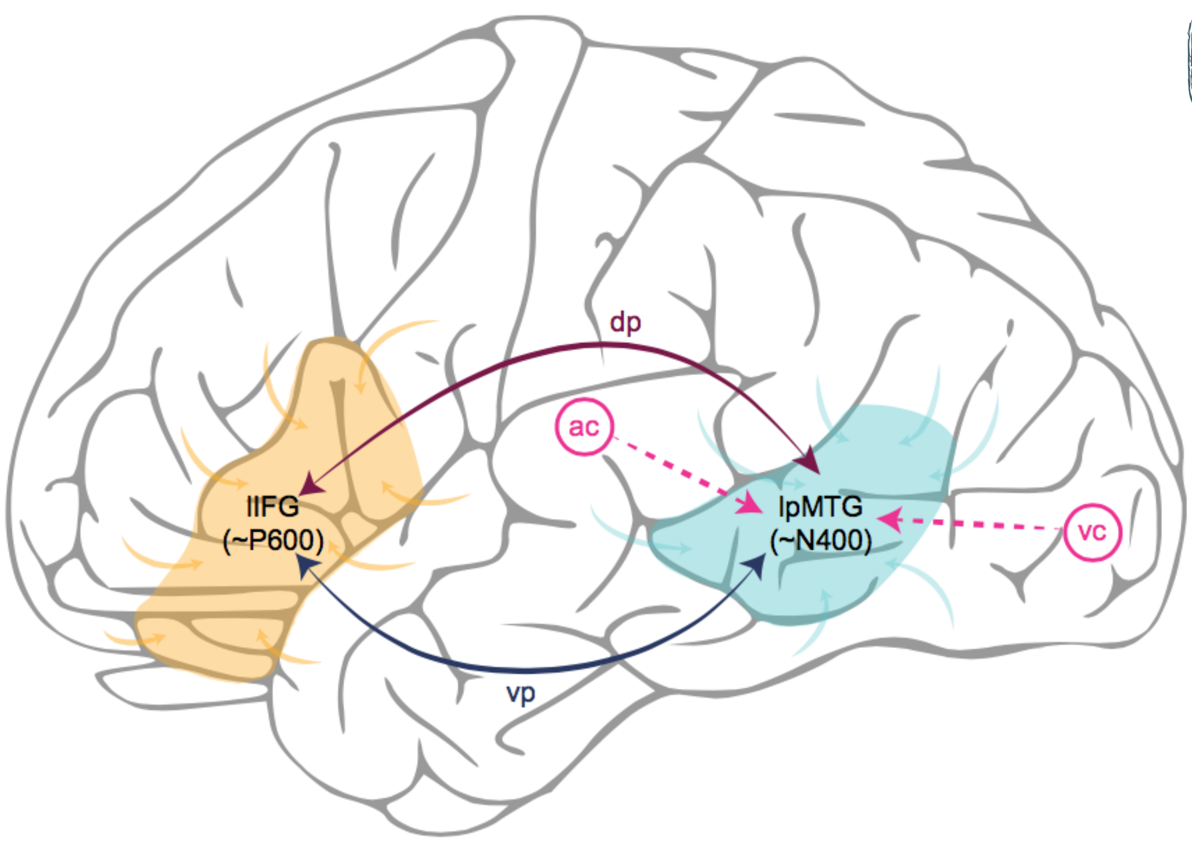
semantic memory	vs.	utterance interpretation
feature-based word meanings		*unfolding discourse-level interpretation*

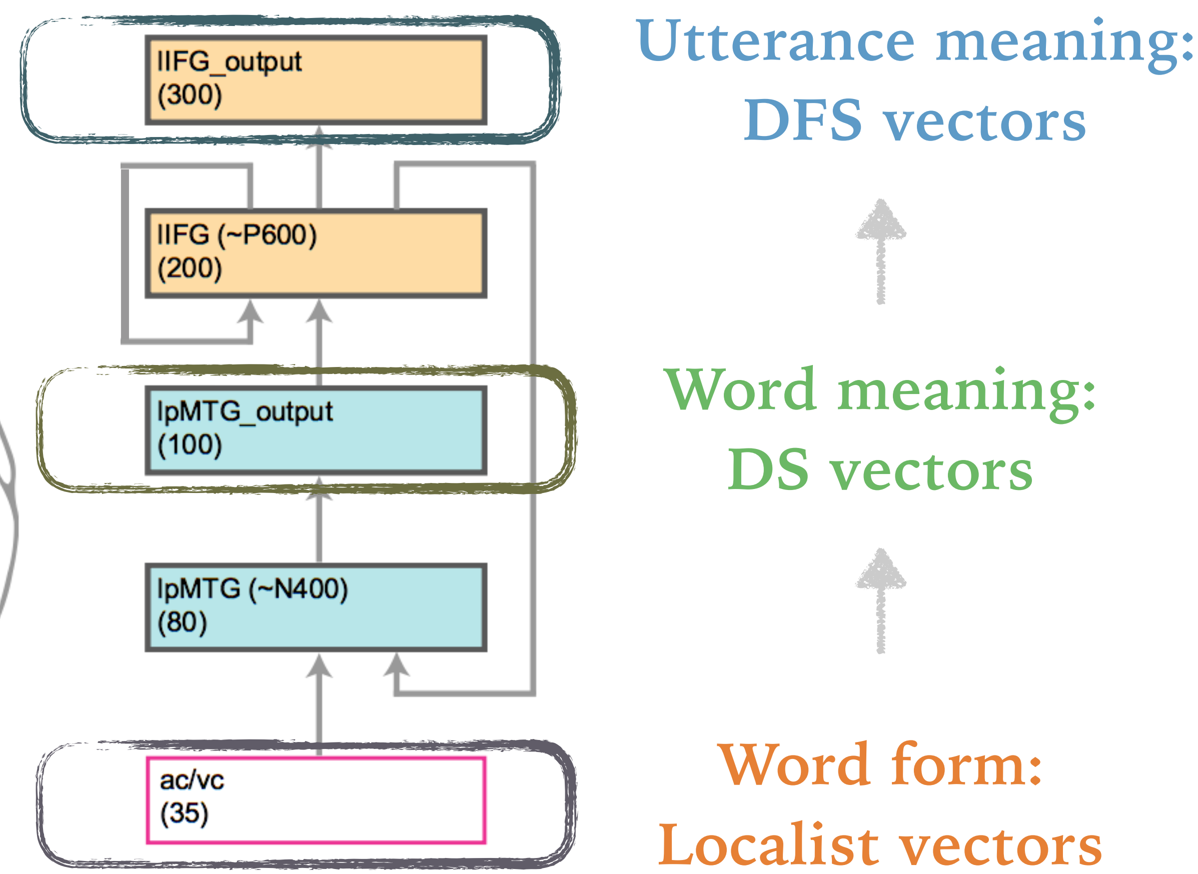
Kutas & Federmeier (2000); McRae et al. (2005); van Berkum (2009); Brouwer et al. (2012, 2017)

DS VS. DFS: A COGNITIVE MODEL OF LANGUAGE COMPREHENSION

The Retrieval-Integration account of the electrophysiology of language comprehension

- Word meaning retrieval~N400
- Integration in utterance meaning~P600



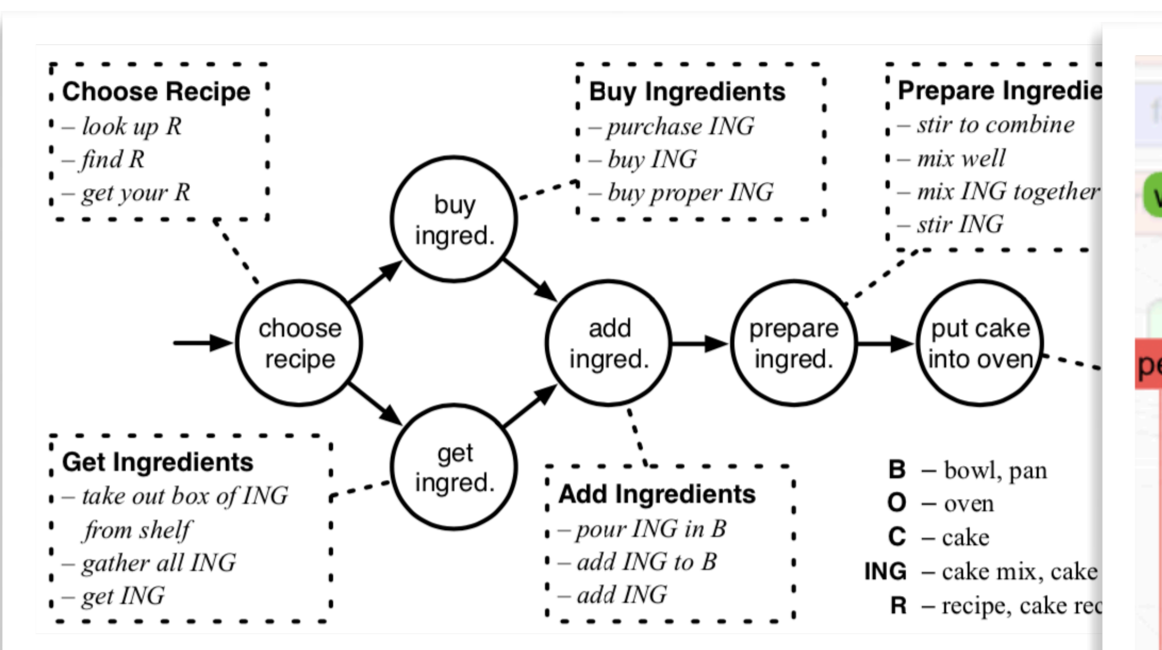


Brouwer et al. (2012, Brain Res.; 2017, Cogn. Sci.; 2021, Frontiers Psych.);

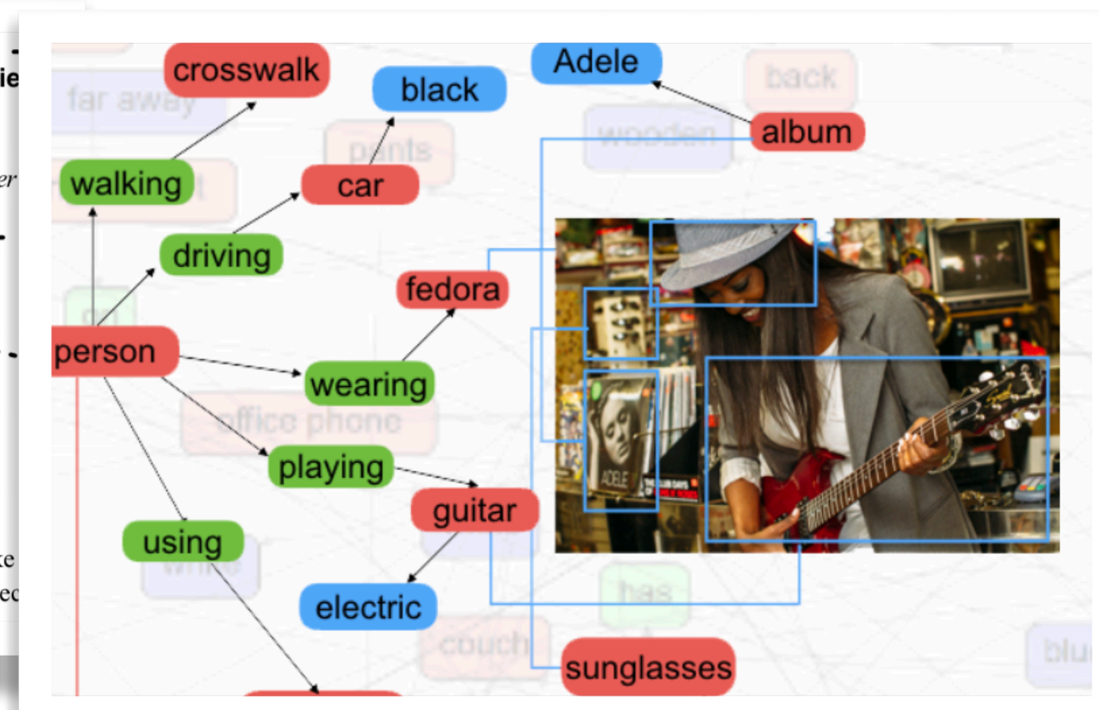
OUTLOOK: DATA-DRIVEN DFS

Goal: Employ DFS to model deep understanding using real-world propositional co-occurrences

- **Step 1**: Derive meaning space from resource with world knowledge about propositional (not: lexical) co-occurrence
- **Step 2**: Mapping natural language to vectors in the meaning space



DeScript corpus (Wanzare et al., 2016)



Visual Genome (Krishna et al., 2016)

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